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LESS STRESS MORE SUCCESS

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Please note:

The Exam questions marked by the symbol in this book are selected from the following:

1. SEC Exam papers
2. Sample exam papers
3. Original and sourced exam-type questions

1 Coordinate Geometry of the Line

aims

- To know where to find the coordinate geometry formulae in the booklet of formulae and tables
- To learn how to apply these formulae to procedural and in-context examination questions
- To gain the ability, with practise, to recall and select the appropriate technique required by the exam questions

Coordinating the plane and plotting points

Coordinates are used to describe the **position** of a point on a plane (flat surface).

Two lines are drawn at right angles to each other.

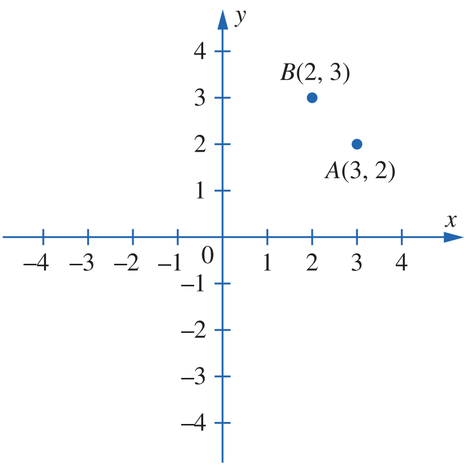
The horizontal line is called the ***x*-axis**.

The vertical line is called the ***y*-axis**.

The two axes meet at a point called the **origin**.

The plane is called the **Cartesian** (kar-tee-zi-an) plane.

Every point on the plane has two coordinates, an ***x*-coordinate** and a ***y*-coordinate**.



The coordinates are enclosed in brackets.

The x-coordinate is always written first, then a comma, followed by the y-coordinate.

On the diagram, the coordinates of the point A are (3, 2).

This is usually written as $A(3, 2)$.

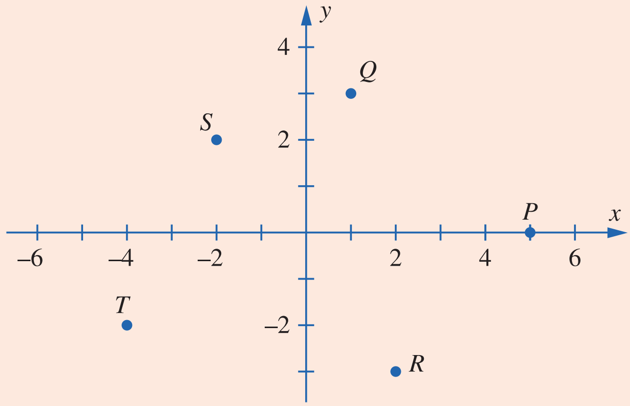
key point

In a couple (x, y) the order is important. The first number, x, is always **across, left** or **right**, and the second number, y, is always **up** or **down**.

The graph above shows the point $A(3, 2)$ is different to the point $B(2, 3)$.

Example

Write down the coordinates of the points P, Q, R, S and T



Solution

$P = (5, 0)$ $Q = (1, 3)$ $R = (2, -3)$ $S = (-2, 2)$ $T = (-4, -2)$

exam Q

An archaeologist has discovered various items at a site. The site is laid out in a grid and the position of each item is shown on the grid. The items found are a brooch (B), a plate (P), a ring (R), a statue (S) and a tile (T).

(a) Write down the coordinates of the position of each item.

$B = ($ 2 , 7 $)$

$P = ($, $)$

$R = ($, $)$

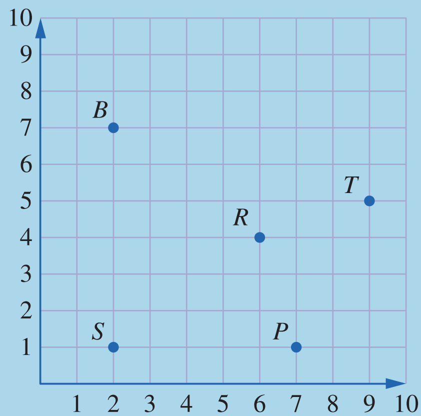
$S = ($, $)$

$T = ($, $)$

(b) Each square of the grid represents 1 m^2. Find the total area of the grid.

(c) Which of the items is nearest to the tile (T)?

(d) Find the distance between the brooch (B) and the statue (S).



Solution

(a) $P = (7, 1)$ $R = (6, 4)$ $S = (2, 1)$ $T = (9, 5)$

(b) Count the grids: 10 up by 10 across
$= 10 \times 10 = 100$ m^2

(c) By observation the ring (R) is nearest to the tile (T).

(d) $B(2, 7)$ to $S(2, 1) = 6$ m

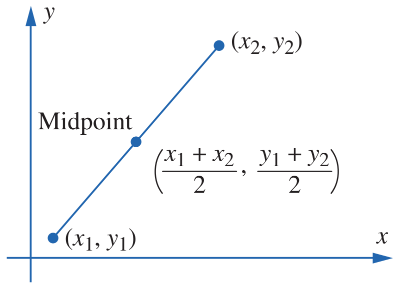
key point

Count each box as one unit.

Midpoint of a line segment

If (x_1, y_1) and (x_2, y_2) are two points, their midpoint is given by the formula:

$$\text{Midpoint} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$



(See booklet of formulae and tables, page 18)

key point

When using coordinate geometry formulae, always allocate one point to be (x_1, y_1) and the other to be (x_2, y_2) before you use the formula.

Example

Noah is positioned at (8, 5) and a bus stop is positioned at (−10, 11). There is a traffic light exactly half way between Noah and the bus stop. Find the coordinates of the traffic light.

Solution

Midpoint (halfway) formula $= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

Let $(x_1, y_1) = (8, 5)$ and $(x_2, y_2) = (-10, 11)$

Coordinates of the traffic light $= \left(\frac{8 - 10}{2}, \frac{5 + 11}{2}\right) = \left(\frac{-2}{2}, \frac{16}{2}\right) = (-1, 8)$

exam Q

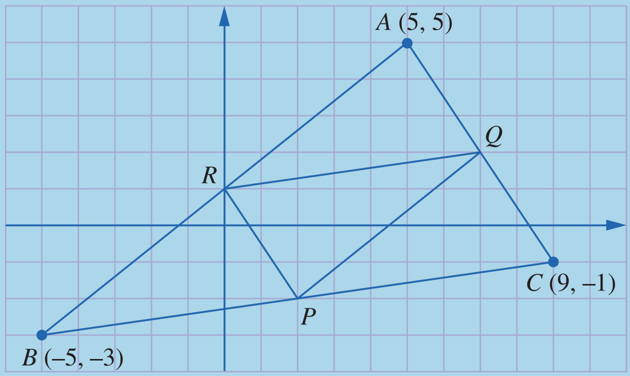
P, *Q* and *R* are the midpoints of the sides of the triangle *ABC*.

(i) Find the coordinates of *P*, *Q* and *R*.

(ii) The number of parallelograms in the diagram is

(a) 0 ☐
(b) 1 ☐
(c) 2 ☐
(d) 3 ☐

Tick the correct answer.



Solution

(i) Use the midpoint formula $\left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$ three times.

Midpoint of [AB]

$(x_1, y_1) = (5, 5)$

$(x_2, y_2) = (-5, -3)$

$R = \left(\dfrac{5-5}{2}, \dfrac{5-3}{2}\right)$

$R = \left(\dfrac{0}{2}, \dfrac{2}{2}\right)$

$R = (0, 1)$

Midpoint of [AC]

$(x_1, y_1) = (5, 5)$

$(x_2, y_2) = (9, -1)$

$Q = \left(\dfrac{5+9}{2}, \dfrac{5-1}{2}\right)$

$Q = \left(\dfrac{14}{2}, \dfrac{4}{2}\right)$

$Q = (7, 2)$

Midpoint of [BC]

$(x_1, y_1) = (-5, -3)$

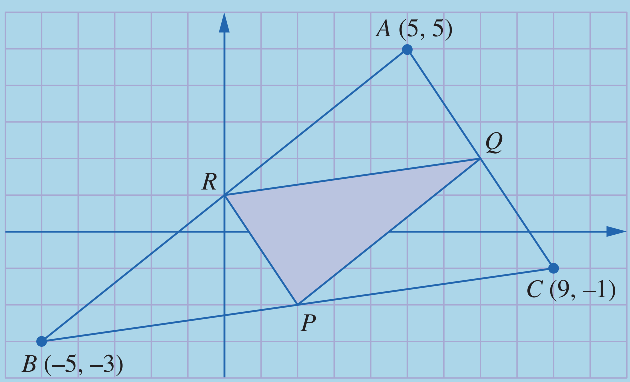
$(x_2, y_2) = (9, -1)$

$P = \left(\dfrac{-5+9}{2}, \dfrac{-3-1}{2}\right)$

$P = \left(\dfrac{4}{2}, \dfrac{-4}{2}\right)$

$P = (2, -2)$

(ii) **(d)** 3 ☑ The shaded triangle in the diagram forms half of three different parallelograms.



Translations

In mathematics, movement in a straight line is called a **translation**.

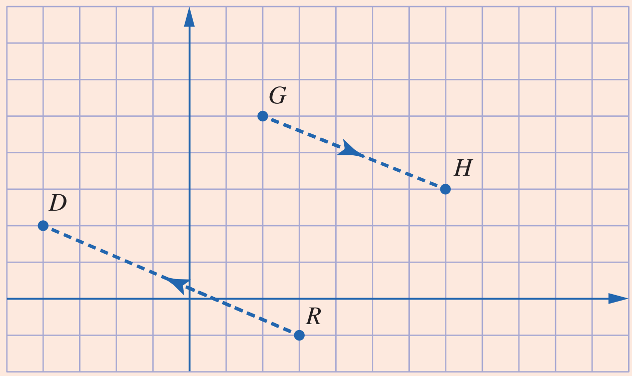
Under a translation, every point is moved the same distance in the same direction. A translation is one of several types of transformations on our course. See Chapter 4 for more on transformations.

Example

Describe the translation that maps the points:

(i) G to H

(ii) R to D



Solution

(i) $G \rightarrow H$ is described by 5 units to the right and 2 units down.

This translation can be written as $\begin{pmatrix} 5 \\ -2 \end{pmatrix}$.

(ii) $R \rightarrow D$ is described by 7 units to the left and 3 units up.

This translation can be written as $\begin{pmatrix} -7 \\ 3 \end{pmatrix}$.

Example

$A\ (-1, 1)$ and $B\ (4, -2)$ are two points.

Find the image of the point $(-1, 3)$ under the translation $\vec{AB}$

Solution

Under the translation $\vec{AB}$: $(-1, 1) \rightarrow (4, -2)$

Rule: Add 5 to x, subtract 3 from y, this can be written as $\begin{pmatrix} 5 \\ -3 \end{pmatrix}$.

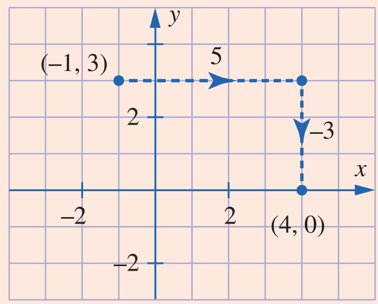
Method 1
Mathematical Method
(Apply the rule directly)

$(-1, 3) \rightarrow (-1 + 5, 3 - 3) = (4, 0)$

We say the image of $(-1, 3)$ is $(4, 0)$.

Method 2
Graphical Method

Plot the point $(-1, 3)$ and split the move into two parts:



- Horizontal move: 5 units to the right (add 5 to x)
- Vertical move: 3 units down (subtract 3 from y)

The image of $(-1, 3)$ is $(4, 0)$.

In some questions, we will be given the midpoint and one end point of a line segment and be asked to find the other end point.

To find the other end point use the following method:

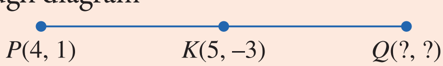
1. Draw a rough diagram.
2. Find the translation that maps (moves) the given end point to the midpoint.
3. Apply the same translation to the midpoint to find the other end point.

Example

If $K\ (5, -3)$ is the midpoint of $[PQ]$ and $P = (4, 1)$, find the coordinates of Q.

Solution

1. Rough diagram



2. Translation from P to K, $\overrightarrow{PK}$.

Rule: add 1 to x, subtract 4 from y. This can be written as $\begin{pmatrix} 1 \\ -4 \end{pmatrix}$.

3. Apply this translation to K:

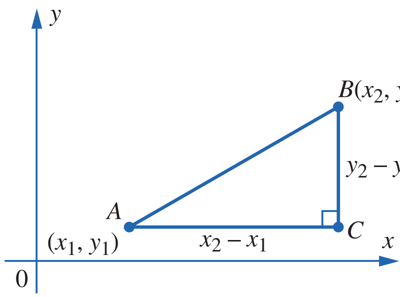
$K(5,-3) \rightarrow (5 + 1, -3 - 4) = (6,-7)$

$\therefore$ The coordinates of Q are $(6, -7)$.

Distance between two points

The given diagram shows the points $A(x_1, y_1)$ and $B(x_2, y_2)$.

$|BC| = y_2 - y_1$ and $|AC| = x_2 - x_1$



Using the theorem of Pythagoras:

$$|AB|^2 = |AC|^2 + |BC|^2$$

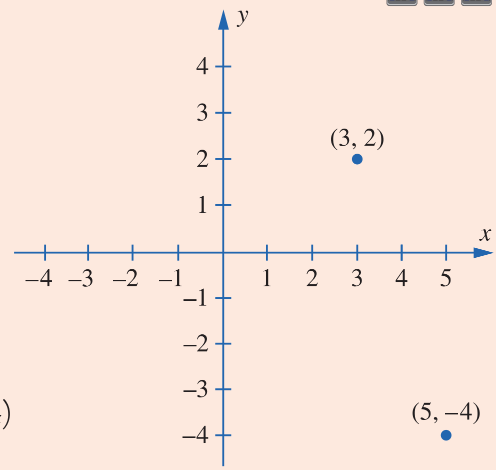
$$|AB|^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$

$$\therefore \quad |AB| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

The distance between $A(x_1, y_1)$ and $B(x_2, y_2)$ is $|AB| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ (see booklet of formulae and tables, page 18).

Example

Carol is positioned at the point (3, 2) and Siobhan is positioned at the point (5, −4). Find the distance between them.



Solution

Let $(x_1, y_1) = (3, 2)$ and $(x_2, y_2) = (5, -4)$

Distance between Carol and Siobhan:

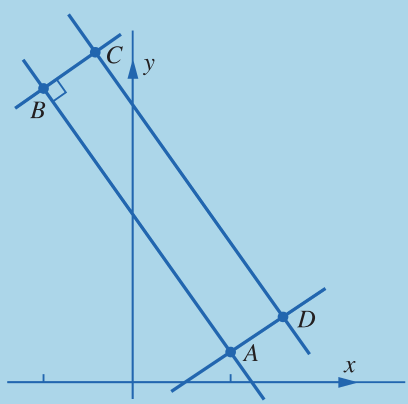
$$= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(5 - 3)^2 + (-4 - 2)^2}$$
$$= \sqrt{(2)^2 + (-6)^2}$$
$$= \sqrt{4 + 36}$$
$$= \sqrt{40}$$
$$= 2\sqrt{10} \quad \text{(using a calculator)}$$

key point

At this stage, all numbers are always positive.

exam Q

ABCD is a rectangle with *A*(3, 1) and *B*(−3, 9). Given $|BC| = \frac{1}{5}|AB|$, calculate the area of *ABCD*.



Solution

Let $(x_1, y_1) = (3, 1)$ and $(x_2, y_2) = (-3, 9)$

$$|AB| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(-3 - 3)^2 + (9 - 1)^2}$$
$$= \sqrt{(-6)^2 + (8)^2} = \sqrt{36 + 64} = \sqrt{100} = 10$$

$$|BC| = \frac{1}{5}|AB| = \frac{1}{5}(10) = 2$$

Area rectangle *ABCD* = (length)(width) = (10)(2) = 20 square units

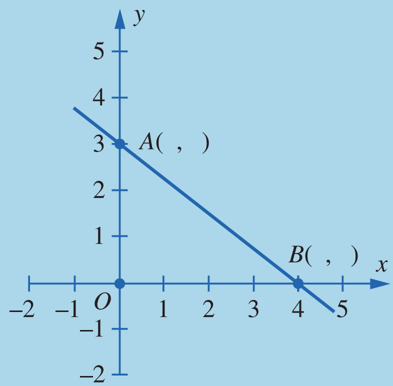
exam Q

(a) Write down the coordinates of the point A and the point B on the diagram.

(b) Use the distance formula to find $|AB|$.

(c) Write down the distance from O to A and the distance from O to B.

(d) Use the theorem of Pythagoras to find the length of the hypotenuse of the triangle OAB.



Solution

(a) $A = (0, 3)$

$B = (4, 0)$

(b) Let $(x_1, y_1) = (0, 3)$ and $(x_2, y_2) = (4, 0)$

$$|AB| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(4 - 0)^2 = (0 - 3)^2}$$
$$= \sqrt{(4)^2 + (-3)^2}$$
$$= \sqrt{16 + 9}$$
$$= \sqrt{25}$$
$$= 5$$

(c) $|OA| = 3$

$|OB| = 4$

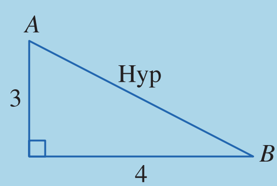
(d) The theorem of Pythagoras

$(\text{Hyp})^2 = (\text{Opp})^2 + (\text{Adj})^2$

$|AB|^2 = (3)^2 + (4)^2$

$|AB|^2 = 9 + 16 = 25$

$|AB| = 5$



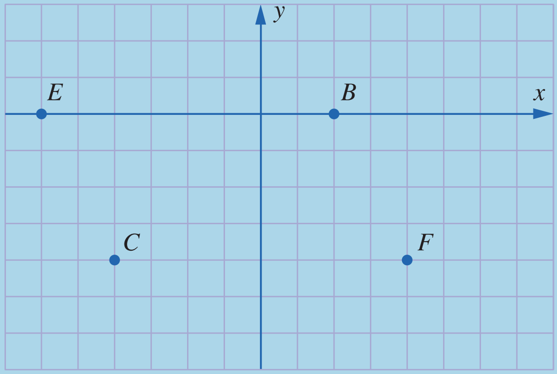
key point

Answer to part (b) = Answer to part (d).

exam Q

Given the points on the diagram:

B	C	E	F
(2, 0)	(−4, −4)	(−6, 0)	(4, −4)



(i) Find:

(a) $|BE|$

(b) $|CF|$

(c) $|EC|$

(d) $|BF|$

(ii) Hence, prove the triangle BCE is congruent to the triangle BCF.

Solution

(i) By observation from the diagram we can say:

(a) $|BE| = 8$

and **(b)** $|CF| = 8$

key point

For parts (a) and (b) count the width of each box as one unit.

We use $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ for both $|EC|$ and $|BF|$.

(c) $|EC|$

$(x_1, y_1) = (-6, 0)$

$(x_2, y_2) = (-4, -4)$

$$|EC| = \sqrt{(-4 - (-6))^2 + (-4 - 0)^2}$$
$$= \sqrt{(-4 + 6)^2 + (-4)^2}$$
$$= \sqrt{(2)^2 + 16}$$
$$= \sqrt{4 + 16}$$
$$= \sqrt{20}$$

(d) $|BF|$

$(x_1, y_1) = (2, 0)$

$(x_2, y_2) = (4, -4)$

$$|BF| = \sqrt{(4 - 2)^2 + (-4 - 0)^2}$$
$$= \sqrt{(2)^2 + (-4)^2}$$
$$= \sqrt{4 + 16}$$
$$= \sqrt{20}$$

key point

$|EC| = |BF|$ and $|BE| = |CF|$. These two pieces of information will be very useful in answering part (ii) of this question.

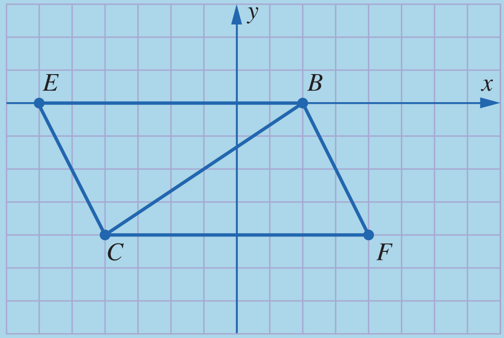
(ii) Consider ΔBCE and ΔBCF

$|BC| = |BC|$ same

$|EC| = |BF| = \sqrt{20}$

$|EB| = |CF| = 8$

Hence (by SSS), ΔBCE is congruent (identical) to ΔBCF.



key point

The four cases for congruent triangles are covered in Chapter 2, Geometry.

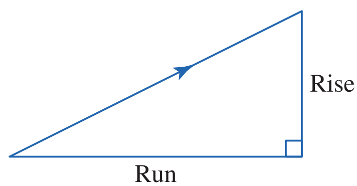
exam focus

- Part **(ii)** above is an excellent example of an exam question linking two different topics on our course. In this case, we see coordinate geometry of the line linked with geometry theorems.
- In a recent exam, a similar question on congruence was asked, but it was worth very few marks. For not answering this part, candidates lost 1 mark out of a total of 27 marks awarded for the question.

Remember: Do not become disheartened, continue to do your best for every part of every question and you will do well.

Slope of a line

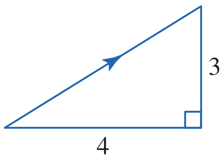
All mathematical graphs are read from **left to right**.
The measure of the steepness of a line is called the **slope**.
The vertical distance, up or down, is called the **rise**.
The horizontal distance across is called the **run**.
The slope of a line is defined as:



$$\text{Slope} = \frac{\text{Rise}}{\text{Run}}$$

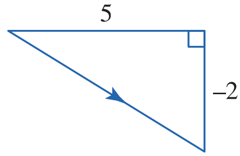
key point

The rise can also be negative and in this case it is often called the '**fall**'. If the rise is zero, then the slope is also zero.



$\text{Slope} = \dfrac{3}{4}$

(Going up)



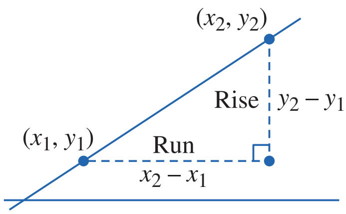
$\text{Slope} = \dfrac{-2}{5}$

(Going down)

$\text{Slope} = \dfrac{0}{6} = 0$

(Horizontal)

Slope of a line containing the points (x_1, y_1) and (x_2, y_2):



If a line contains two points (x_1, y_1) and (x_2, y_2) then the slope is given by the formula:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

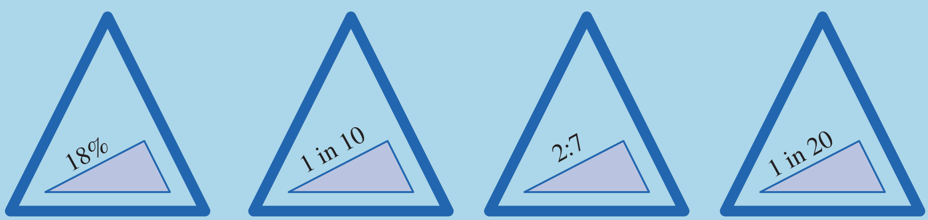
(see page 18 in the booklet of formulae and tables)

exam Q

In a certain country, the slope of a hill is defined as $\dfrac{\text{Rise}}{\text{Run}}$.

Which hill is the steepest? Which hill is the least steep? Justify your answers.

(i) **(ii)** **(iii)** **(iv)**



Solution

(i) $18\% = \dfrac{18}{100} = 0{\cdot}18$

(ii) $1 \text{ in } 10 = \dfrac{1}{10} = 0{\cdot}1$

(iii) $\dfrac{2}{7} = 0{\cdot}2857$

(iv) $\dfrac{1}{20} = 0{\cdot}05$

By observing the above four calculations we conclude:

Hill number **(iii)** has the greatest slope and so it is the steepest.

Hill number **(iv)** has the smallest slope and so it is the least steep.

The work above is my justification.

key point

Converting to decimals is the easiest way to compare the slopes.

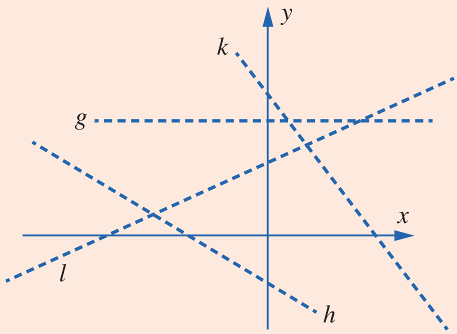
Example

Which of the lines g, h, k, l in the diagram has:

(i) A slope of zero?

(ii) A positive slope?

Justify your answers.



Solution

(i) By observation, line g makes no angle with the x-axis (it is horizontal)
$\Rightarrow g$ has a slope of zero.

(ii) Reading the diagram from left to right, we observe line l is going up
$\Rightarrow l$ has a positive slope.

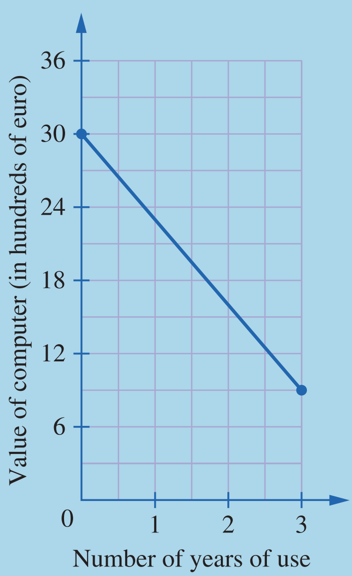
exam focus

Some exam solutions may be short and wordy.

exam Q

An accountant plots the value of a computer over a three-year period on the given graph. Find the average rate of change.

Interpret your answer in the context of the question.



Solution

key point

The average rate of change = m = the slope of the line

Method 1

$$m = \frac{\text{Rise}}{\text{Run}} = \frac{\text{down from 30 to 9}}{\text{3 years}}$$

$$= \frac{-21}{3} = -7$$

Method 2

$(x_1, y_1) = (0, 30)$ and $(x_2, y_2) = (3, 9)$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{9 - 30}{3 - 0} = \frac{-21}{3} = -7$$

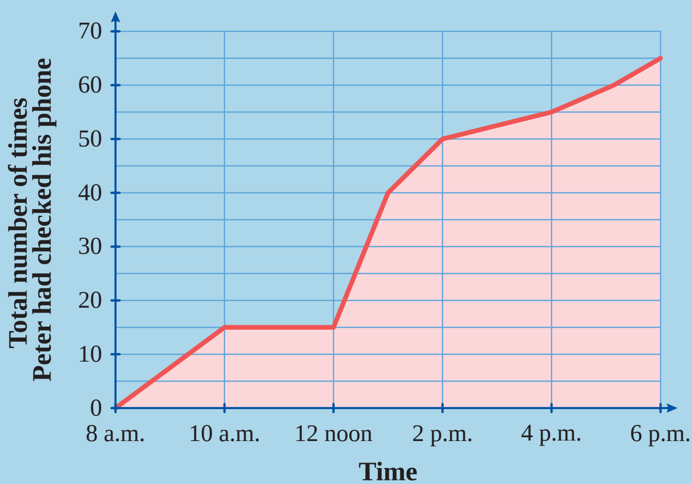
In the context of this question, a slope of $-7(=-€700)$ indicates the rate of change in the value of the computer each year.

exam focus

Either method will gain full marks in this case.

exam Q

The graph below shows the total number of times Peter checked his phone from 8 a.m. to 6 p.m. on a given day. For example, by 6 p.m. Peter had checked his phone a total of 65 times.



(a) Use the graph to answer each of the following questions. In each case, tick (✓) the correct box only.

(i) By 2 p.m., the total number of times Peter had checked his phone was:

15	40	50	55
☐	☐	☐	☐

(ii) Peter did not check his phone at all from:

10 – 12 noon	12 – 2 p.m.	2 – 4 p.m.	4 – 6 p.m.
☐	☐	☐	☐

(iii) Peter checked his phone most often from:

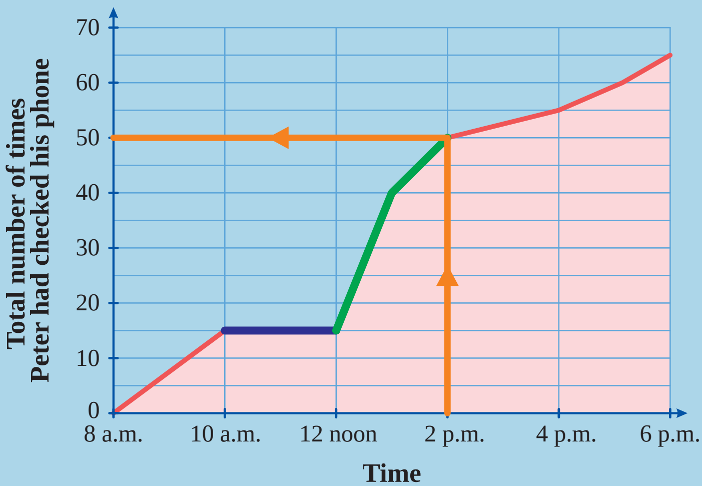
10 – 12 noon	12 – 2 p.m.	2 – 4 p.m.	4 – 6 p.m.
☐	☐	☐	☐

(b) From 8 a.m. to 6 p.m. on that day, Peter checked his phone on average 6·5 times each hour. He uses this to estimate N, the total number of times he had checked his phone, as: $N = 6{\cdot}5 \times H$
where H is the number of hours after 8 a.m. on that day.

(i) Use this formula to find the value of N when H is 8.

(ii) Peter uses his formula to estimate that he will have checked his phone 156 times by 8 a.m. the following day (when $H = 24$). Do you think that this is a reasonable estimate?

Solution



(a) **(i)** From the graph (orange arrow), we can see that at 2 p.m., Peter has checked his phone 50 times.

15	40	50	55
☐	☐	☑	☐

(ii) The part of the graph where Peter didn't check his phone is represented by a horizontal line (coloured in blue).
This happened between 10 a.m. and 12 noon.

10 – 12 noon	12 – 2 p.m.	2 – 4 p.m.	4 – 6 p.m.
☑	☐	☐	☐

(iii) The part of the graph where Peter checked his phone most often is represented by the steepest section (coloured in green).
This happened between 12 noon and 2 p.m.

10 – 12 noon	12 – 2 p.m.	2 – 4 p.m.	4 – 6 p.m.
☐	☑	☐	☐

(b) (i) Let $H = 8$:

$$N = 6{\cdot}5 \times H$$
$$N = 6{\cdot}5 \times 8$$
$$N = 52$$

exam focus

Being able to substitute values in for variables is a vital skill for you to have. You will use this skill quite often as you work through the exam paper.

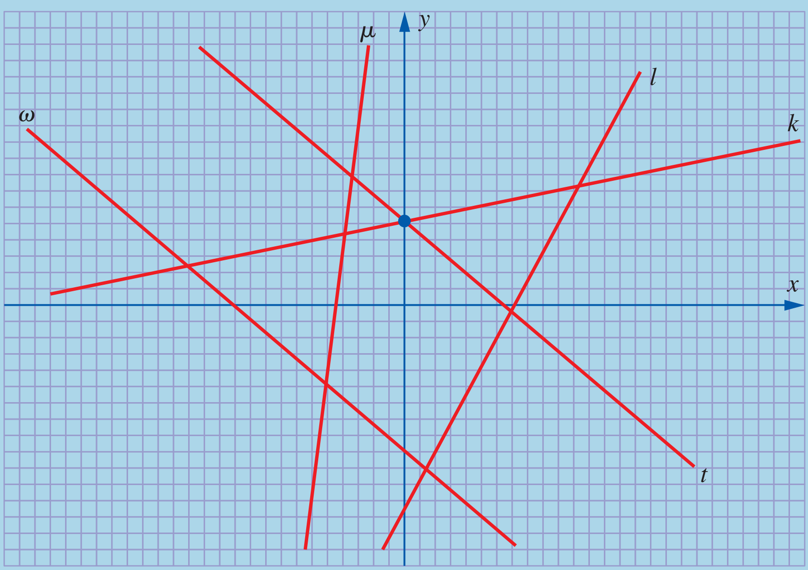
(ii) This formula is based on Peter's day-time usage between 8 a.m. and 6 p.m. (10 hours duration).
It is unlikely that his usage will be the same for the other 14 hours of the 24-hour period. For example, Peter will not be checking his phone while he is asleep.
So, the estimate of 156 times is probably a bit too high and we can conclude that it is not a reasonable estimate.

Parallel lines

To prove whether or not two lines are parallel, do the following:

1. Find the slope of each line.
2. (a) If the slopes are the same, the lines are parallel.
(b) If the slopes are different, the lines are **not** parallel.

exam Q



Five lines μ, ω, t, l and k in the coordinate plane are shown in the diagram above.

The slopes of the five lines are given in the table.

Complete the table, matching the lines to their slopes.

Slope	Line
$\frac{1}{6}$	
$\frac{5}{3}$	
$-\frac{9}{10}$	
13	
$-\frac{9}{10}$	

key point

Two lines have slope $-\frac{9}{10}$. This means two lines are parallel.

$\therefore$ t has slope $-\frac{9}{10}$ and ω has slope $-\frac{9}{10}$.

Solution

l, k and μ all have positive slopes (because they are all rising).

By observation, μ has the steepest positive slope.
$\therefore$ μ has slope 13.

Also by observation, k has the least steep positive slope.

$\therefore$ k has slope $\frac{1}{6}$.

Since the only remaining line is l and the only remaining slope is $\frac{5}{3} \Rightarrow$ l has slope $\frac{5}{3}$.

Slope	Line
$\frac{1}{6}$	k
$\frac{5}{3}$	
$-\frac{9}{10}$	t
13	μ
$-\frac{9}{10}$	ω

The equation of a line

The formula: $y - y_1 = m(x - x_1)$ (see booklet of formulae and tables, page 18)

gives the equation of a line when we have:

- A point on the line (x_1, y_1)
- The slope of the line, m.

Example

Find the equation of the line through the point $(5, -1)$ whose slope is $\frac{2}{3}$.

Solution

$y - y_1 = m(x - x_1)$

$(x_1, y_1) = (5, -1)$ and $m = \frac{2}{3}$

$$\therefore \quad y - (-1) = \frac{2}{3}(x - 5)$$

$$y + 1 = \frac{2}{3}(x - 5)$$

$$3(y + 1) = 2(x - 5) \quad \text{(multiply both sides by 3 to remove the fraction)}$$

$$3y + 3 = 2x - 10$$

$$3y = 2x - 10 - 3$$

$$3y = 2x - 13$$

exam Q

P is the point $(-1, 7)$ and Q is the point $(1, -3)$. Find the equation of the line PQ.

key point

The slope is missing. We first find the slope and then use **either** point to find the equation.

Solution

$$\left.\begin{array}{l} P(-1, 7) = (x_1, y_1) \\ Q(1, -3) = (x_2, y_2) \end{array}\right\} \quad \text{Slope} = m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-3 - 7}{1 - (-1)} = \frac{-10}{1 + 1} = \frac{-10}{2} = -5$$

Equation of PQ with slope $= m = -5$ and $Q = (1, -3) = (x_1, y_1)$

$$y - y_1 = m(x - x_1)$$

$$y - (-3) = -5(x - 1)$$

$$y + 3 = -5x + 5$$

$$y = -5x + 5 - 3$$

$$y = -5x + 2$$

The slope of a line when given its equation

To find the slope of a line when given its equation, do the following:

> Get y on its own, and the number in front of x is the slope.

Note: The number in front of x is called the **coefficient** of x.
The number on its own is called the y **intercept**.

In short: write the line in the form $y = mx + c$.

$$y = \underset{\downarrow}{mx} + \underset{\downarrow}{c}$$ (see booklet of formulae and tables, page 18)

$$y = (\text{slope})x + (\text{where the line cuts the } y\text{-axis})$$

Example

Write down the slope, m, of each of the following lines.

(i) $y = 4x - 3$ **(ii)** $y = 8 - 2x$ **(iii)** $y = x + 5$

(iv) $2y = 7x - 10$ **(v)** $y - 6x = 0$ **(vi)** $3y + 2x + 12 = 0$

Solution

Using $y = mx + c$ in each case:

(i) $y = 4x - 3 \Rightarrow m = 4$

(ii) $y = 8 - 2x \Rightarrow m = -2$ (be careful to include the minus)

(iii) $y = x + 5 \Rightarrow m = 1$ (not zero)

(iv) $2y = 7x - 10$ (divide each term by 2 to get)

$$y = \frac{7}{2}x - 5 \Rightarrow m = \frac{7}{2}$$

(v) $y - 6x = 0$

$$y = 6x \Rightarrow m = 6$$

(vi) $3y + 2x + 12 = 0$

$$3y = -2x - 12$$ (divide each term by 3 to get)

$$y = \frac{-2}{3}x - 4 \Rightarrow m = -\frac{2}{3}$$

exam Q

k is the line $2x - y + 5 = 0$.

Find the equation of the line that is parallel to k and passes through the point $(-1, 4)$.

Solution

Find the slope of the line k:

$$2x - y + 5 = 0$$

$$\text{Slope} = -\frac{\text{Number in front of } x}{\text{Number in front of } y}$$

$$\text{Slope} = -\frac{2}{-1}$$

$$\text{Slope} = 2$$

Since k is parallel to the required line, the slope of the required line is $+2$.

Find the equation of the line with slope, $m = 2$ and passes through the point $(-1, 4) = (x_1, y_1)$:

$$y - y_1 = m(x - x_1)$$

$$y - 4 = 2(x - (-1))$$

$$y - 4 = 2(x + 1)$$

$$y - 4 = 2x + 2$$

$$0 = 2x - y + 2 + 4$$

$$0 = 2x - y + 6$$

To verify that a point belongs to a line

To verify that a point belongs to a line, substitute the coordinates of the point into the equation of the line. If the coordinates satisfy the equation, then the point is on the line. Otherwise, the point is not on the line.

Example

Investigate if the points $(-2, 9)$ and $(-5, 3)$ are on the line $5x - 3y + 34 = 0$.

Solution

$(-2, 9)$ $\quad 5x - 3y + 34 = 0$

Substitute $x = -2$ and $y = 9$

$$5(-2) - 3(9) + 34$$

$$= -10 - 27 + 34$$

$$= -37 + 34$$

$$= -3 \neq 0$$

Does not satisfy the equation

$\therefore$ $(-2, 9)$ is not on the line.

$(-5, 3)$ $\quad 5x - 3y + 34 = 0$

Substitute $x = -5$ and $y = 3$

$$5(-5) - 3(3) + 34$$

$$= -25 - 9 + 34$$

$$= -34 + 34$$

$$= 0$$

Satisfies the equation

$\therefore$ $(-5, 3)$ is on the line.

Example

(i) The point $(k, -2)$ is on the line $4x + 3y - 14 = 0$. Find the value of k.
(ii) The point $(1, 2)$ is on the line $3x + ty - 11 = 0$. Find the value of t.

Solution

(i) $4x + 3y - 14 = 0$

Substitute $x = k$ and $y = -2$

$4(k) + 3(-2) - 14 = 0$

$4k - 6 - 14 = 0$

$4k - 20 = 0$

$4k = 20$

$k = 5$

(ii) $3x + ty - 11 = 0$

Substitute $x = 1$ and $y = 2$

$3(1) + t(2) - 11 = 0$

$3 + 2t - 11 = 0$

$2t - 8 = 0$

$2t = 8$

$t = 4$

Graphing lines

To draw a line, we need only two points. The easiest points to find are where lines cut the x- and y-axes. This is known as the **intercept method**.

key point

On the x-axis, $y = 0$. On the y-axis, $x = 0$.

To draw a line, do the following:

1. Let $y = 0$ and find x.
2. Let $x = 0$ and find y.
3. Plot these two points.
4. Draw the line through these points.

If the constant in the equation of a line is zero, e.g. $3x - 5y = 0$, or $4x = 3y$, then the line will pass through the origin, $(0, 0)$. In this case the **intercept method** will not work.

To draw a line that contains the origin, $(0, 0)$, do the following:

1. Choose a suitable value for x and find the corresponding value for y (or vice versa).
2. Plot this point.
3. A line drawn through this point and the origin is the required line.

key point

One method that usually works is to let x equal the number in front of y and then find the corresponding value for y (or vice versa).

Example

Graph the line $3x + 4y = 0$.

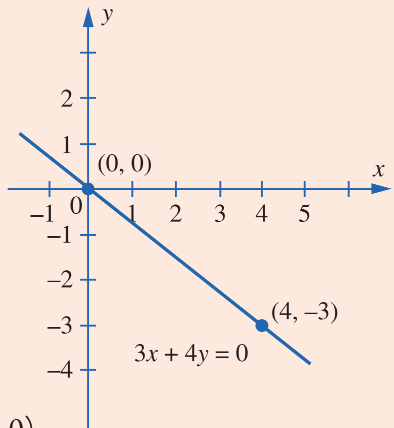
Solution

1. Let $x = 4$ (number in front of y).

$$3x + 4y = 0$$
$$\downarrow$$
$$3(4) + 4y = 0$$
$$12 + 4y = 0$$
$$4y = -12$$
$$y = -3$$

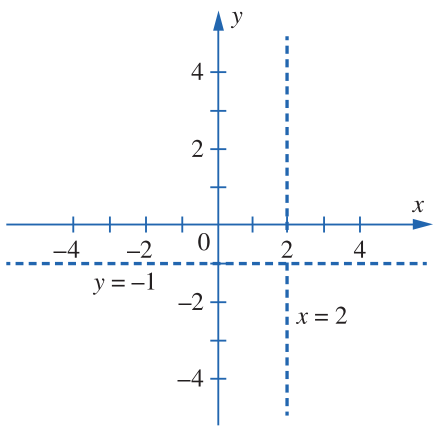
2. Plot the point $(4, -3)$.

3. Draw the line through the points $(4, -3)$ and $(0, 0)$.



Lines parallel to the axes

$x = 2$ is a line parallel to the y-axis through 2 on the x-axis.
$y = -1$ is a line parallel to the x-axis through -1 on the y-axis.



key point

$y = 0$ is the equation of the x-axis.
$x = 0$ is the equation of the y-axis.

exam focus

All horizontal lines (parallel to x-axis) have an angle of inclination of 0°, which shows their slopes are zero.
All vertical lines (parallel to y-axis) have an angle of inclination of 90°, which shows their slopes are infinitely steep.

exam Q

p is the line $5x - 6y + 30 = 0$.

p cuts the x-axis at A and the y-axis at B.

(i) Find the coordinates of A and B.

(ii) Draw the graph of p.

(iii) Find the area of the triangle OAB, where O is the origin.

Solution

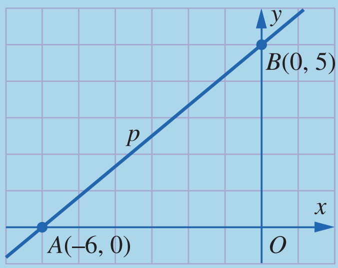
(i) On the x-axis, $y = 0$.

On the y-axis, $x = 0$.

$5x - 6y = -30$

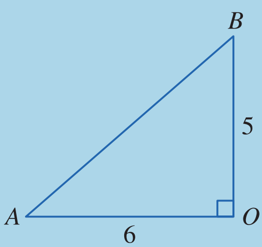
$y = 0$	$x = 0$
$5x - 0 = -30$	$0 - 6y = -30$
$x = -6$	$6y = 30$
$(-6, 0)$	$y = 5$
	$(0, 5)$

(ii)



The coordinates are $A(-6, 0)$ and $B(0, 5)$.

(iii)



Area of triangle OAB

$= \frac{1}{2}$ (base) (perpendicular height)

$= \frac{1}{2}(6)(5)$

$= 15$

exam Q

k is the line $2x - 5y + 15 = 0$.

(i) Find the slope of k.

(ii) k cuts the x-axis at the point Q. Find the coordinates of the point Q.

Solution

(i) $2x - 5y + 15 = 0$

$-5y = -2x - 15$

$5y = 2x + 15$ (multiply each term by -1)

$$y = \frac{2}{5}x + \frac{15}{5}$$

$\therefore$ The slope of the line $k = \frac{2}{5}$. (using $y = mx + c$)

(ii) On the x-axis $y = 0$, hence, we put $y = 0$ into

$$2x - 5y + 15 = 0$$
$$2x - 5(0) + 15 = 0$$
$$2x - 0 + 15 = 0$$
$$2x + 15 = 0$$
$$2x = -15$$
$$x = -\frac{15}{2}$$

When $y = 0$, $x = -\frac{15}{2}$.

The line k cuts the x-axis at the point $Q\left(-\frac{15}{2}, 0\right)$.

exam focus

Sometimes the answers to challenging questions may include fractions.

exam Q

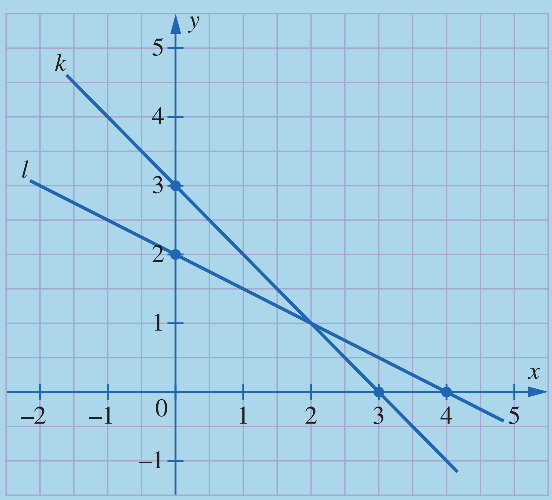
$l: x + 2y = 4$ and $k: x + y = 3$ are the equations of two lines shown on the diagram.

(i) From the graph, write down the point of intersection of l and k.

(ii) Solve algebraically the simultaneous equations

$$x + 2y = 4$$
$$x + y = 3$$

and verify your answer to part **(i)**.



Solution

(i) From the graph the point of intersection is (2, 1).

(ii) Label the equations ① and ②.

$$x + 2y = 4 \quad ①$$
$$x + y = 3 \quad ②$$

Make the coefficients of x the same but of opposite signs.

Leave ① unchanged, multiply ② by –1.

$$x + 2y = 4 \quad ①$$
$$-x - y = -3 \quad ② \times -1$$

Add these new equations.

$$y = 1$$

Put $y = 1$ into ① or ②

$$x + y = 3 \quad ②$$
$$\downarrow$$
$$x + 1 = 3$$
$$x = 2$$

key point

More examples on solving simultaneous equations may be found in *Less Stress More Success Maths Book 1.*

Therefore the point of intersection of L and K is (2, 1), which is the same answer as in part (i).

13 Classroom-Based Assessments (CBAs)

aims

- ☐ To become familiar with the four elements of assessment for Junior Cycle Mathematics
- ☐ To be familiar with the details of the Classroom-Based Assessment 2
- ☐ To be able to understand and apply the Statistical-Enquiry Cycle
- ☐ To be familiar with the criteria of quality for assessment
- ☐ To understand the four descriptors for the CBA and the criteria associated with each descriptor
- ☐ To understand the steps involved in starting your investigation and examining a menu of suggestions for investigation
- ☐ To be familiar with the procedure involved with how to carry out a statistical investigation
- ☐ To be able to use the checklist provided to ensure that you haven't missed any key elements in your investigation

Introduction

As mentioned in the Introduction chapter of this book, your assessment in Junior Cycle Mathematics consists of four elements.

1. **Classroom-Based Assessment 1 (CBA 1)**

   This is a mathematical investigation and it is carried out during your second year of the three-year Junior Cycle. **CBA 1 is covered in *Less Stress More Success Maths Book 1*.**

2. **Classroom-Based Assessment 2 (CBA 2)**

   This is a statistical investigation and it is carried out during your third year of the three-year Junior Cycle. **CBA 2 is covered in this chapter.**

3. **Assessment Task**

   This is a written assignment and it is carried out during your third year of the three-year Junior Cycle, after you have completed CBA 2.

4. **Written exam paper**

   This is a 2-hour written exam and it take place at the end of third year, with the rest of your written exams.

CBA 2: Statistical Investigation

The investigation is an opportunity for you to show that you can apply statistics to an area that interests you. Your teacher will give you a timetable and deadline for submitting your investigation.

The details of the investigation are as follows:

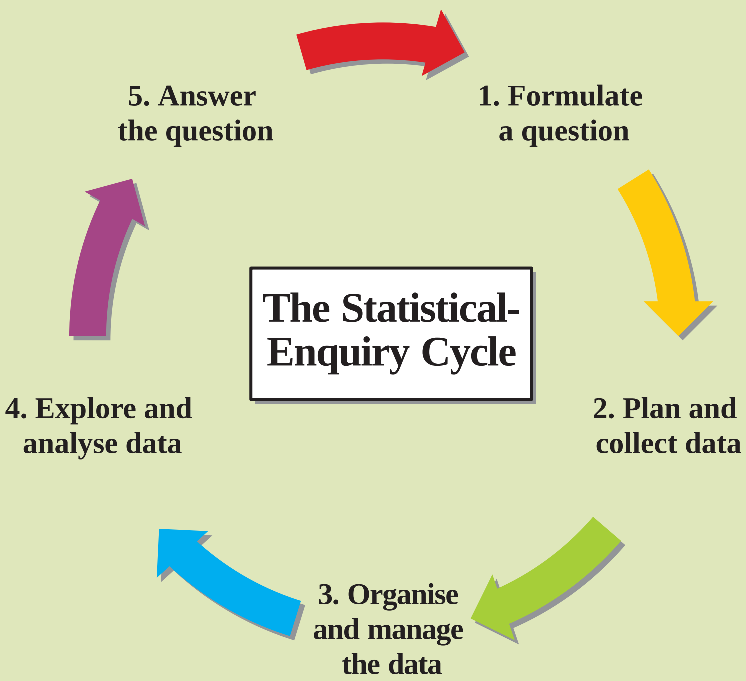
Format: A report may be presented in a wide range of formats.
Preparation: A student will, over a three-week period in third year, follow the Statistical-Enquiry Cycle to investigate a mathematical problem.

The Statistical-Enquiry Cycle is as follows:

1. Formulate a question
2. Plan and collect unbiased, representative data
3. Organise and manage the data
4. Explore and analyse the data, using appropriate displays and numerical summaries
5. Answer the original question, giving reasons based on the analysis section

key point

You must be familiar with the Statistical-Enquiry Cycle:



CBA 2: Assessment criteria and four descriptors

The investigation is assessed by the class teacher. A student will be awarded one of the following categories of achievement:

- Yet to meet expectations
- In line with expectations
- Above expectations
- Exceptional

Assessment criteria

A good investigation should be clear and easily understood by one of your fellow classmates (peers) and self-explanatory all of the way through.

The criteria are split into four areas A, B, C and D:

A. Designing the investigation
B. Identifying the variables of interest
C. Organising and managing the data
D. Analysing and interpreting data summaries

Linking the criteria with the four categories of achievement (descriptors)

A. Designing the investigation

Criteria	Achievement
Uses given statistics question and collection method	Yet to achieve expectations
Poses a question that anticipates variability and plans to collect/source the type of data appropriate for the question posed	In line with expectations
Poses a question that anticipates variability and seeks generalisation; data collection plan shows awareness of how variability affects the validity and reliability of the findings	Above expectations
Poses a question that anticipates variability and seeks generalisation, study design will produce as far as practical reliable and valid results by taking into account variability and confounding variables	Exceptional

B. Identifying the variables of interest

Criteria	Achievement
Gathers and displays data	Yet to achieve expectations
Identifies variable and develops a measuring strategy for measuring the dependent and independent variable	In line with expectations
Chosen measuring strategy provides valid and reliable data	Above expectations
Describes relationship between the variables and describes considerations related to reliability and fairness	Exceptional

C. Organising and managing the data

Criteria	Achievement
Makes statements about the data displayed	Yet to achieve expectations
Displays data in a way that allows patterns to be identified; identifies patterns and describes the data in terms of those patterns	In line with expectations
Uses appropriate data displays and describes the data in terms of measures of centre and spread	Above expectations
Uses distributions to analyse the data and justifies measures of centre used to describe the data	Exceptional

D. Analysing and interpreting data summaries

Criteria	Achievement
No concrete connection back to the original question	Yet to achieve expectations
Makes a concrete connection to the original question of the investigation but does not look beyond the data	In line with expectations
Reports the findings and the conclusion refers to the original question and attempts to look beyond the data	Above expectations
Interprets the data in relation to the original question; conclusion displays understanding of the limitations of generalising to the population and considers the need to reformulate the original question in light of the findings	Exceptional

Academic honesty

Academic honesty means that your work is based on your own original ideas and not copied from other people. However, you may draw on the work and ideas of others, but this must be acknowledged. This would be put into a reference list at the end of your investigation, known as a bibliography. In addition, you should use your own language and expression.

Record-keeping

Throughout the investigation, keep a journal, either on paper or online. This journal will also help you to demonstrate academic honesty. The journal will be of great assistance in focusing your efforts when writing your CBA 2 investigation.

- Make notes of any websites or books you use
- You are encouraged to use a variety of support materials and present your work in a variety of formats
- Keep a record of your actions so you can show your teacher how much time you are spending on your investigation
- Remember to follow your teacher's advice and meet your CBA 2 timetable
- The teacher is there to facilitate you, so do not be afraid to ask for guidance. The more focused your questions are, the better guidance your teacher can give you.

Evidence of learning

The following evidence is required

- A report
- Student research records

You must report your research and findings in a format of your choice. The report can be completed at the end of the investigation. If a typed or hand-written report is the format of choice, the total length of the report would typically be in the 650–800 words range (excluding tables, graphs, reference list and research records), but this should not be regarded as a rigid requirement.

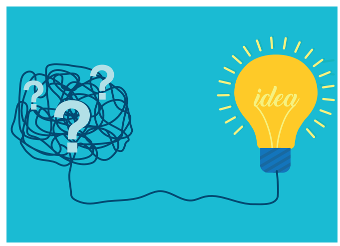
A statistical investigation may be presented in other formats, quite effectively (e.g. posters, podcasts or multimedia). However, you must take care that all the research can be judged on the final product alone. For example, a poster presentation may allow you to select and present highlights of your research, but it is also necessary to include a written report of approximately 400 words to show the deeper research carried out.

Vital tools for the Statistical Investigation

The following tools should prove very useful to you when carrying out your Statistical Investigation:

- The three chapters in this book
  - o Statistics I: Statistical Investigations
  - o Statistics II: Central Tendency and Spread of Data
  - o Statistics III: Representing Data
- Two pages on 'Glossary of Statistical Terms' at the end of this chapter
- *Census at schools* website, which has a large store of recorded data. This could help you to prove or disprove your assertions
- Be familiar with appropriate use of technology to sort and display data (e.g. spreadsheets)
- Highlight the data points that belong to you in your displays (if appropriate)

Choosing a topic



You should choose a topic that you are interested in, because then you will be inclined to put more effort into the project. In addition, you will enjoy working on your project and this will shine through. You should discuss the topic with your teacher before you put too much time and effort into it, in case your idea is not in line with what a Statistical Investigations should be.

If you cannot think of a topic yourself, then you can ask your teacher for help in coming up with a topic to investigate. Below are some ideas that might help you to come up with an investigation of your own.

Suggestions for investigation, with ideas to consider:

- Investigate eating trends of today's youths.
  - o Sample group
  - o Survey on eating habits
  - o Vegetarians?
  - o Fruit or vegetable intake
  - o Sugar intake