CONTENTS

LESS STRESS MORE SUCCESS

Please note:

The exam questions marked by the symbol exam Q in this book are selected from the following:

1. SEC exam papers
2. Sample exam papers
3. Original and sourced exam-type questions

1 Number Systems

aims

- ☐ To learn what the symbols $\mathbb{N}$, $\mathbb{Z}$, $\mathbb{Q}$, and $\mathbb{R}$ represent
- ☐ To be familiar with prime numbers, factors and the fundamental theorem of arithmetic
- ☐ To be able to find LCM and HCF as required

Natural numbers $\mathbb{N}$

The positive whole numbers 1, 2, 3, 4, 5, 6, . . . are also called the counting numbers. The dots indicate that the numbers go on forever and have no end (infinite).

exam Q

Give two reasons why $-7{\cdot}3$ is not a natural number.

Solution

Reason 1. It is a negative number

Reason 2. It is not a whole number (it is a decimal)

Factors (divisors)

key point

The factors of any whole number are the whole numbers that divide exactly into the given number, leaving no remainder.

1 is a factor of every number.

Every number is a factor of itself.

Example

Find the highest common factor of 18 and 45.

Solution

18	45
1×18	1×45
2×9	3×15
3×6	5×9

The common factors are 1, 3 and 9.

The highest common factor of 18 and 45 is 9.

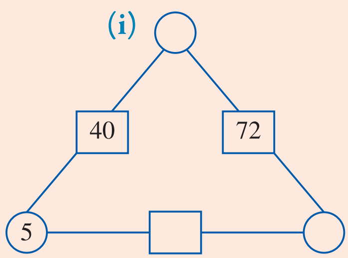
key point

The highest common factor of two or more numbers is the largest factor that is common to each of the given numbers.

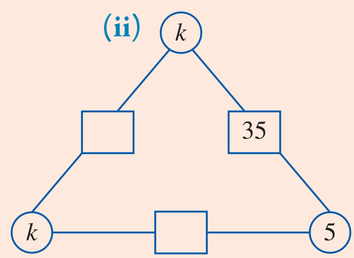
Example

In these productogons, the number in each square is the product of the numbers in the circles on each side of it. Find the missing numbers in each of these productogons.

(i)



(ii)

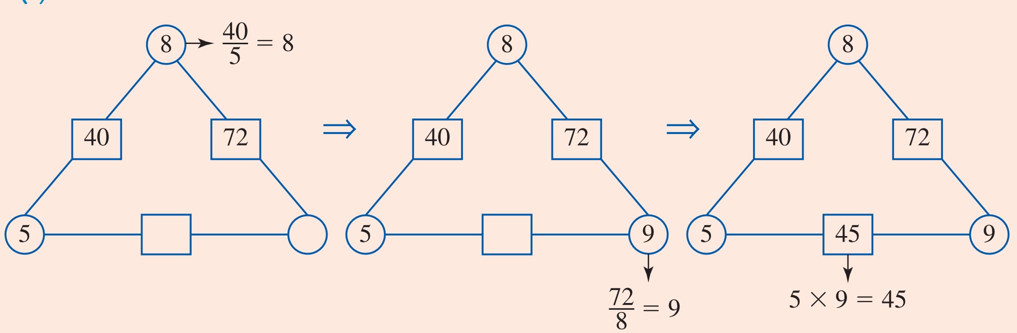


Solution

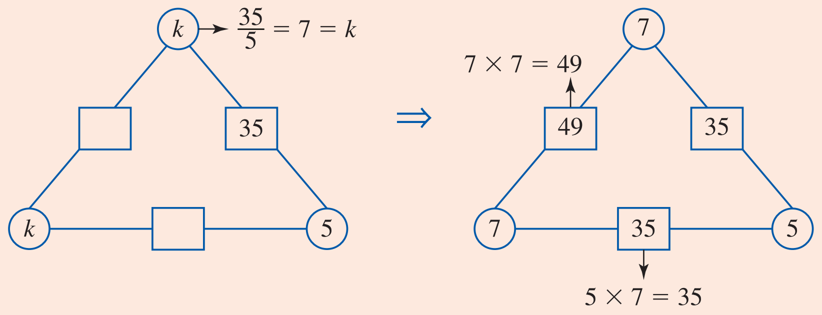
key point

The use of the word productogon in the question indicates we use multiplication. This is because product means multiply.

(i)



(ii)



Prime numbers

key point

A prime number is a whole number greater than 1 that has only two factors, 1 and itself.

The first 12 prime numbers are

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31 and 37.

There is an infinite number of prime numbers.

Numbers that have more than two factors are called composite numbers, e.g. 20 has 1, 2, 4, 5, 10, 20 as factors.

exam focus

The fundamental theorem of arithmetic states that any whole number greater than 1 can be written as the product of its prime factors in a **unique** way.
This will underpin many exam questions on number theory.

Prime factors

Any number can be expressed as a product of prime numbers. To express the number 180 as a product of its prime numbers, first divide by the smallest prime number that will divide exactly into it.

The smallest prime number 2	:	$2 \,\vert\, 180$
The smallest prime 2 again	:	$2 \,\vert\, 90$
The smallest prime 3	:	$3 \,\vert\, 45$
The smallest prime 3 again	:	$3 \,\vert\, 15$
The smallest prime 5	:	$5 \,\vert\, 5$
		1

So 180 expressed as a product of primes is:

$$2 \times 2 \times 3 \times 3 \times 5 = 2^2 \times 3^2 \times 5$$

Example 1

For security, a credit card is encrypted using prime factors. A huge number is assigned to each individual card and it can only be verified by its prime factor decomposition. Find the 10-digit natural number which is assigned to the following credit cards whose prime factor decomposition is

(i) $2^2 \times 11 \times 13 \times 17^2 \times 19^3$

(ii) $2^7 \times 3^2 \times 5^2 \times 7^3 \times 23 \times 31$

Solution

By calculator: (i) 1133847572

(ii) 7043299200

Example 2

Gepetto makes wooden puppets. He has three lengths of wood which he wants to cut into pieces, all of which must be the same length and be as long as possible. The lengths of the three pieces of wood are 315 cm, 357 cm and 252 cm.

(i) Express each of the three lengths as a product of primes.

(ii) Hence, calculate what length each piece should be and how many pieces he will have.

Solution

(i)

$$\begin{array}{r|r} 3 & 315 \\ \hline 3 & 105 \\ \hline 5 & 35 \\ \hline 7 & 7 \\ \hline & 1 \end{array} \qquad \begin{array}{r|r} 3 & 357 \\ \hline 7 & 119 \\ \hline 17 & 17 \\ \hline & 1 \end{array} \qquad \begin{array}{r|r} 2 & 252 \\ \hline 2 & 126 \\ \hline 3 & 63 \\ \hline 3 & 21 \\ \hline 7 & 7 \\ \hline & 1 \end{array}$$

$3^2 \times 5 \times 7$ $\qquad$ $3 \times 7 \times 17$ $\qquad$ $2^2 \times 3^2 \times 7$

(ii) By observation of the three product of primes above, the highest common factor (HCF) is given by $3 \times 7 = 21$.

Hence, each piece of wood should be 21 cm long.

The number of pieces is given by $\dfrac{315}{21} + \dfrac{357}{21} + \dfrac{252}{21}$

$= 15 + 17 + 12$

$= 44$

key point

3×7 is common to all three lengths.

Multiples and the lowest common multiple (LCM)

The multiples of a number are found by multiplying the number by 1, 2, 3 . . . and so on.

The multiples of 4 are: 4, 8, 12, 16, 20, . . .

The multiples of 7 are: 7, 14, 21, 28, 35, . . .

The **lowest common multiple** of two or more numbers is the **smallest multiple** that is common to each of the numbers.

In other words, the lowest common multiple is the **smallest** number into which each of the numbers will divide exactly.

For example, the lowest common multiple of 2, 4 and 7 is 28, as 28 is the smallest number into which 2, 4 and 7 will all divide exactly.

The lowest common multiple of two or more numbers is found with the following steps:

1. Write down the multiples of each number.
2. The lowest common multiple is the smallest (first) multiple they have in common.

Example

K is the set of natural numbers from 1 to 25, inclusive.

(i) List the elements of K that are multiples of 3.
(ii) List the elements of K that are multiples of 5.
(iii) Write down the lowest common multiple of 3 and 5.

Solution

K = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25}

(i) Multiples of 3 = {3, 6, 9, 12, 15, 18, 21, 24}
(ii) Multiples of 5 = {5, 10, 15, 20, 25}
(iii) Lowest common multiple (LCM) is 15.
That is the smallest number that both sets have in common.

Integers $\mathbb{Z}$

Negative numbers are numbers below zero. Positive and negative **whole** numbers, including zero, are called integers.
Integers can be represented on a number line:

Integers to the right of zero are called **positive integers**.

Integers to the left of zero are called **negative integers**.

Example

At midnight on Christmas Eve the temperatures in some cities were as shown in the table.

New York	2°C
Rome	−2°C
Dublin	−1°C
Moscow	−20°C
Cairo	4°C

(i) Which city recorded the

  (a) Lowest temperature

  (b) Highest temperature?

(ii) List the temperatures from coldest to hottest.

(iii) Which cities had a temperature difference of 6°C?

(iv) What is the difference in temperature between

  (a) Dublin and Moscow

  (b) Cairo and Dublin?

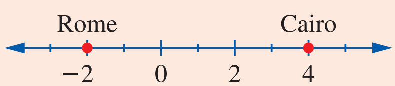
Solution

(i) (a) Lowest temperature, $-20°$, in Moscow.

  (b) Highest temperature, $4°$, in Cairo.

(ii) $-20, -2, -1, 2, 4$

(iii)



Rome and Cairo have a difference of 6°C

(iv) (a) Dublin and Moscow $= -1 - (-20) = -1 + 20 = 19°$

  (b) Cairo and Dublin $= 4 - (-1) = 4 + 1 = 5°$

Fractions (rational numbers)

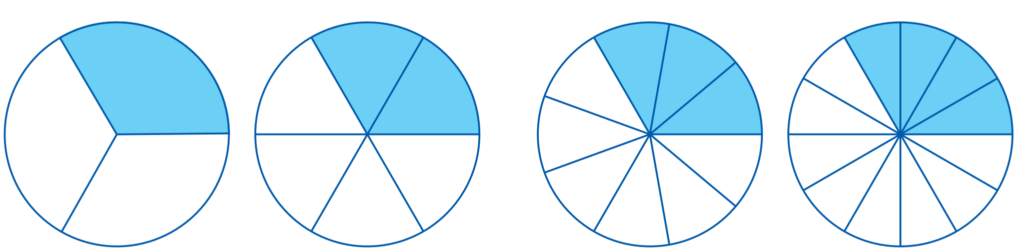
A fraction is written as two whole numbers, one over the other, separated by a bar.
Equivalent fractions are fractions that are equal.
For example:

$$\frac{1}{3} = \frac{2}{6} = \frac{3}{9} = \frac{4}{12}$$

This can be shown on a diagram where the same proportion is shaded in each circle.

key point

$$\text{Fraction} = \frac{\text{Numerator}}{\text{Denominator}}$$



$\frac{1}{3} = \frac{2}{6} = \frac{3}{9} = \frac{4}{12}$

A rational number (fraction) is a number that can be written as a ratio, $\frac{p}{q}$, of two integers, p and q, but $q \neq 0$.

Examples are $\frac{7}{2}$, $-\frac{11}{19}$, $8 = \frac{8}{1}$, $0 = \frac{0}{1}$, $5{\cdot}23 = \frac{523}{100}$

Rational numbers are denoted by the letter $\mathbb{Q}$.

Example

Bren is trying to subtract $\frac{1}{5}$ from $\frac{7}{8}$.

His attempt is shown here: $\frac{7}{8} - \frac{1}{5} = \frac{6}{3} = 2$.

(i) Explain what Bren has done wrong.

(ii) Write out the correct solution.

Solution

(i) It seems that Bren has subtracted the top numbers and subtracted the bottom numbers.

key point

For subtraction or addition of fractions, we must find a common denominator.

(ii) $\frac{7}{8} - \frac{1}{5} \Rightarrow$ common denominator $= 8 \times 5 = 40$

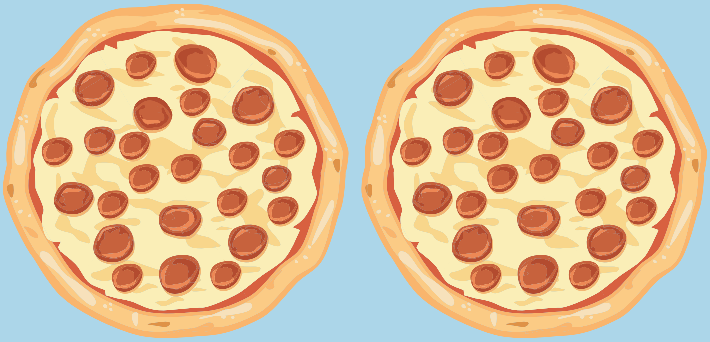
Then $\frac{7}{8} - \frac{1}{5} = \frac{(5)(7) - (8)(1)}{40} = \frac{35 - 8}{40} = \frac{27}{40}$.

key point

See the chapter on algebraic expressions for more on addition/subtraction of fractions.

exam Q

Sheila orders two pizzas to divide evenly between herself and five friends.



(i) What fraction of a pizza will each person get? Write your fraction in its simplest form.

(ii) One of the friends gets a text and leaves before the pizza is delivered. What fraction will each person now get if the pizzas are divided evenly between those remaining?

(iii) Find how much extra pizza each person gets.

Solution

(i) 2 pizzas $\div$ 6 $= \frac{2}{6} = \frac{1}{3}$ each

(ii) 2 pizzas $\div$ 5 $= \frac{2}{5}$ each

(iii) Extra pizza $= \frac{2}{5} - \frac{1}{3}$

$= \frac{(2)(3) - (1)(5)}{15}$ (common denominator $= 5 \times 3 = 15$)

$= \frac{6 - 5}{15}$

$= \frac{1}{15}$

exam Q

Three students completed a test but got their results in different ways. The teacher told Karen that she got 0·7 of the questions correct. John was told he got 80% of the questions correct. David was told he got $\frac{3}{4}$ of the questions correct.

(i) Which student got the best result? Give a reason for your answer.

(ii) There were 20 questions on the test. How many questions each did Karen, John and David answer correctly?

(iii) Find the mean number of correct answers.

Solution

(i) Karen got $0{\cdot}7 = \frac{7}{10} = \frac{7 \times 100}{10}\% = 70\%$

John got 80%

David got $\frac{3}{4} = \frac{3 \times 100}{4}\% = 75\%$

By observation from the above work, John got the best result.

(ii) Karen got 70% of 20 $= \frac{70}{100} \times 20 = 14$ correct

John got 80% of 20 $= \frac{80}{100} \times 20 = 16$ correct

David got 75% of 20 $= \frac{75}{100} \times 20 = 15$ correct

(iii) Mean $= \frac{14 + 16 + 15}{3} = \frac{45}{3} = 15$

key point

Mean is covered in Statistics, in Book 2.

exam focus

The question was awarded 20 marks in total, as follows.

Part **i** 10 marks, with 5 marks awarded for one correct piece of work.

Part **ii** 5 marks, with 3 marks awarded for one correct piece of work.

Part **iii** 5 marks, with 3 marks awarded for one correct piece of work.

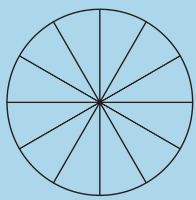
Hence, with 11 marks out of 20 marks awarded for no correct answers, you can see the importance of attempting every part of every question.

(a) Find the value of each of the following

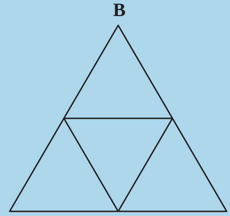
(i) $372 + 119$ (ii) $3{\cdot}4 \times 7$ (iii) $3 \times (7 - 5)$

(b) Shade in $\frac{3}{4}$ of the area of each shape. The shapes are labelled A and B.

A



B



(c) Write the numbers **3**, **9**, and **25** into the three empty boxes to make the mathematical statement true. Use each number only once.

$$\frac{\square}{5} + \frac{\square}{\square} = \frac{24}{25}$$

Solution

(a) You can use a calculator to find the value of each part:

(i) $372 + 119$
$= 491$

(ii) $3{\cdot}4 \times 7$
$= 23{\cdot}8$

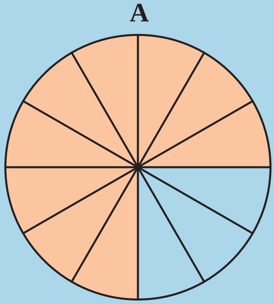
(iii) $3 \times (7 - 5)$
$= 3 \times 2$ (simplify inside brackets first)
$= 6$

(b) Shape A is divided into 12 equal parts.

$$\frac{3}{4} \text{ of } 12 = 9$$

So, we need to shade in 9 of the 12 sectors:

A

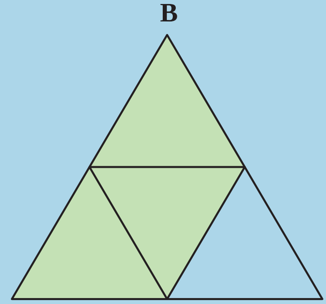


Shape B is divided into 4 equal parts.

$$\frac{3}{4} \text{ of } 4 = 3$$

So, we need to shade in 3 of the 4 triangles:

B



(c) Since the final result has a denominator of 25, the original denominators must both be factors of 25.

So, we can conclude that 25 goes into the position of the missing denominator.

Since the final result of $\frac{24}{25}$ is less than 1, in each case, the numerator must be less than the denominators. So, 3 goes above the 5 and 9 goes above the 25.

$$\frac{\boxed{3}}{\boxed{5}} + \frac{\boxed{9}}{\boxed{25}} = \frac{\boxed{24}}{\boxed{25}}$$

key point

You can use your calculator to verify the answer

$\frac{3}{5} + \frac{9}{25}$ $\frac{24}{25}$

exam Q

(i) Write $\frac{3}{8}$ as a decimal.

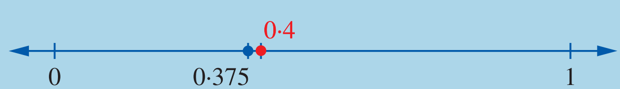
(ii) Represent the numbers $\frac{3}{8}$ and 0·4 on the number line below.

(iii) How could the number line in **(ii)** above help you decide which is the bigger of the two numbers?

Solution

(i) $\frac{3}{8} = 0{\cdot}375$

(ii)



(iii) The number to the right-hand side on a number line is always the bigger of the two numbers.

0·4 is the bigger number.

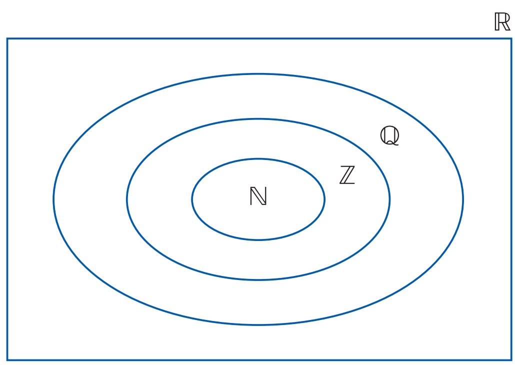
Real numbers

The natural numbers ($\mathbb{N}$) are a subset of the integers ($\mathbb{Z}$).
The integers ($\mathbb{Z}$) are a subset of the rational numbers ($\mathbb{Q}$).
The rational numbers ($\mathbb{Q}$) are a subset of the real numbers ($\mathbb{R}$).

exam focus

The number sets are in the formulae and tables booklet, which is available to you in the examination.

The following Venn diagram summarises the number system.



See the chapter on sets, where the first exam question illustrates the way the examiner links sets and the number system.

key point

The numbers outside the set $\mathbb{Q}$ are called irrational numbers. You will study irrational numbers at Leaving Certificate.

exam Q

The columns in the table below represent the following sets of numbers: natural numbers ($\mathbb{N}$), integers ($\mathbb{Z}$), rational numbers ($\mathbb{Q}$). Complete the table by writing either YES or NO into each box, indicating whether each of the numbers -23, $\frac{1}{2}$, 7^2, 2·95, 0, 47 is or is not an element of each.

(One box has already been filled in. The YES indicates that the number 47 is an element of the set of rational numbers, $\mathbb{Q}$.)

Set \ Number	-23	$\frac{1}{2}$	7^2	2·95	0	47
$\mathbb{N}$						
$\mathbb{Z}$						
$\mathbb{Q}$						Yes

Solution

key point

$7^2 = 49$ and 47 are the only natural numbers, $\mathbb{N}$, in the question. 49, 47, 0 and -23 are the only integers, $\mathbb{Z}$, and all six numbers are rational, $\mathbb{Q}$.

Set \ Number	-23	$\frac{1}{2}$	7^2	2·95	0	47
$\mathbb{N}$	No	No	Yes	No	No	Yes
$\mathbb{Z}$	Yes	No	Yes	No	Yes	Yes
$\mathbb{Q}$	Yes	Yes	Yes	Yes	Yes	Yes

17 Classroom-Based Assessments (CBAs)

aims

- ☐ To become familiar with the four elements of assessment for Junior Cycle mathematics
- ☐ To be familiar with the details of the Classroom-Based Assessment 1
- ☐ To be able to understand and apply the Problem-Solving Cycle
- ☐ To be familiar with the criteria of quality for assessment
- ☐ To understand the four descriptors for the CBA and the criteria associated with each descriptor
- ☐ To understand the steps involved in starting your investigation and examining a menu of suggestions for investigation
- ☐ To be familiar with the procedure involved with how to carry out a mathematical investigation
- ☐ To be able to use the checklist provided to ensure that you haven't missed any key elements in your investigation

Introduction

As mentioned in the Introduction chapter of this book, your assessment in Junior Cycle mathematics consists of four elements.

1. **Classroom-Based Assessment 1 (CBA 1)**

   This is a mathematical investigation and it is carried out during your second year of the three-year Junior Cycle. **CBA 1 is covered in this chapter.**

2. **Classroom-Based Assessment 2 (CBA 2)**

   This is a statistical investigation and it is carried out during your third year of the three-year Junior Cycle. **CBA 2 is covered in the *Less Stress More Success Maths Book 2*.**

3. **Assessment Task**

   This is a written assignment and it is carried out during your third year of the three-year Junior Cycle, after you have completed CBA 2.

4. **Written exam paper**

   This is a 2-hour written exam and it take place at the end of third year, with the rest of your written exams.

CBA 1: Mathematical Investigation

The investigation is an opportunity for you to show that you can apply mathematics to an area that interests you. Your teacher will give you a timetable and deadline for submitting your investigation.

The details of the investigation are as follows:

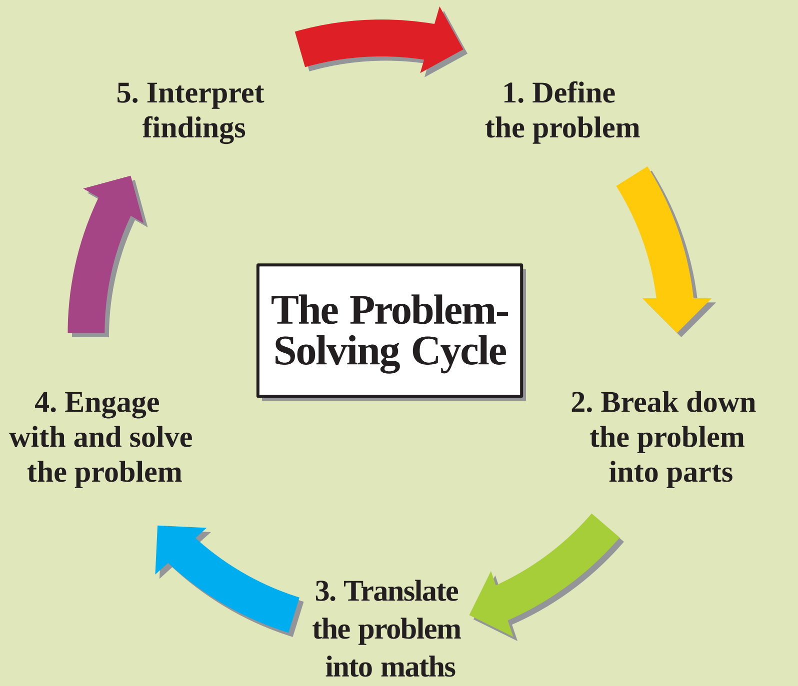
Format: A report may be presented in a wide range of formats.
Preparation: A student will, over a three-week period in second year, follow the Problem-Solving Cycle to investigate a mathematical problem.

The Problem-Solving Cycle is as follows:

1. Define a problem
2. Decompose it into manageable parts and/or simplify it using appropriate assumptions
3. Translate the problem to mathematics, if necessary
4. Engage with the problem and solve it, if possible
5. Interpret any findings in the context of the original problem

key point

You must be familiar with the Problem-Solving Cycle:



CBA 1: Assessment criteria and four descriptors

The investigation is assessed by the class teacher. A student will be awarded one of the following categories of achievement:

- Yet to meet expectations
- In line with expectations
- Above expectations
- Exceptional

Assessment criteria

A good investigation should be clear and easily understood by one of your fellow classmates (peers) and self-explanatory all of the way through.

The criteria are split into four areas A, B, C and D:

- **A.** Defining the problem statement
- **B.** Finding a strategy or translating the problem to mathematics
- **C.** Engaging with the mathematics to solve the problem
- **D.** Interpreting and reporting

Linking the criteria with the four categories of achievement (descriptors)

A. Defining the problem statement

Criteria	Achievement
Uses a given problem statement and with guidance breaks the problem down into steps	Yet to achieve expectations
With guidance poses a problem statement, breaks the problem down into manageable steps and simplifies the problem by making assumptions, if appropriate	In line with expectations
With limited guidance poses a problem statement and clarifies/simplifies the problem by making reasonable assumptions, where appropriate	Above expectations
Poses a concise problem statement and clarifies and simplifies the problem by making justified assumptions, where appropriate	Exceptional

B. Finding a strategy or translating the problem to mathematics

Criteria	Achievement
Uses a given strategy	Yet to achieve expectations
Chooses an appropriate strategy to engage with the problem	In line with expectations
Justifies the use of a suitable strategy to engage with the problem and identifies any relevant variables	Above expectations
Develops an efficient justified strategy and evaluates progress towards a solution where appropriate; conjectures relationship between variables where appropriate	Exceptional

C. Engaging with the mathematics to solve the problem

Criteria	Achievement
Records some observations/data and follows some basic mathematical procedures	Yet to achieve expectations
Records observations/data and follows suitable mathematical procedures with minor errors; graphs and/or diagrams/words are used to provide insights into the problem and/or solution	In line with expectations
Records observations/data systematically, suitable mathematical procedures are followed, and accurate mathematical language, symbolic notation and visual representations are used; attempts are made to generalise any observed patterns in the solution/observation	Above expectations
Mathematical procedures are followed with a high level of precision, and a justified answer is achieved; solution/observations are generalised and extended to other situations where appropriate	Exceptional

D. Interpreting and reporting

Criteria	Achievement
Comments on any solution	Yet to achieve expectations
Comments on the reasonableness of the solution where appropriate and makes a concrete connection to the original question, uses everyday familiar language to communicate ideas	In line with expectations
Checks reasonableness of solution and revisits assumptions and/or strategy to iterate the process, if necessary, uses formal mathematical language to communicate ideas and identifies what worked well and what could be improved	Above expectations
Deductive arguments used and precise mathematical language and symbolic notation used to consolidate mathematical thinking and justify decisions and solutions; strengths and/or weaknesses in the mathematical representation/solution strategy are identified	Exceptional

Academic honesty

Academic honesty means that your work is based on your own original ideas and not copied from other people. However, you may draw on the work and ideas of others, but this must be acknowledged. This would be put into a reference list at the end of your investigation, known as a bibliography. In addition, you should use your own language and expression.

Record-keeping

Throughout the investigation, keep a journal, either on paper or online. This journal will also help you to demonstrate academic honesty. The journal will be of great assistance in focusing your efforts when writing your CBA 1 investigation.

- Make notes of any websites or books you use.
- You are encouraged to use a variety of support materials and present your work in a variety of formats.
- Keep a record of your actions so you can show your teacher how much time you are spending on your investigation.
- Remember to follow your teacher's advice and meet your CBA 1 timetable.
- The teacher is there to facilitate you, so do not be afraid to ask for guidance. The more focused your questions are, the better guidance your teacher can give you.

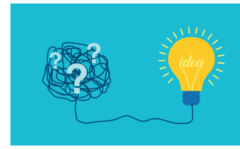
Evidence of learning

The following evidence is required

- A report
- Student research records

You must report your research and findings in a format of your choice. The report can be completed at the end of the investigation. If a typed or hand-written report is the format of choice, the total length of the report would typically be in the 400–600 words range (excluding tables, graphs, reference list and research records), but this should not be regarded as a rigid requirement.

Choosing a topic



You should choose a topic that you are interested in, because then you will be inclined to put more effort into the project. In addition, you will enjoy working on your project and this will shine through. You should discuss the topic with your teacher before you put too much time and effort into it, in case your idea is not in line with what a mathematical investigation should be.

If you cannot think of a topic yourself, then you can ask your teacher for help in coming up with a topic to investigate. Below are some ideas that might help you to come up with an investigation of your own.

Suggestions for investigation, with ideas to consider:

- Investigating the cost of a family weekend in a foreign city
    - Destination
    - Transportation
    - Hotel
    - Currency exchange
    - Activities
- Garden design
    - Size, shape and dimensions of garden
    - Features: flower bed/pond/trees/patio
    - Draw a sketch
    - Work out costs
- Bedroom makeover
    - Carpet/wooden floor
    - Walls painted or wallpapered
    - Furniture – bed/desk/wardrobe
    - Decoration
    - Work out costs
- Best placement of security sensors in different shaped rooms
    - Radius of detection on the average sensor
    - Best placement in a rectangular room/square room/L-shaped room
- An environmental investigation, for example, glass recycling
    - Mass of glass per bottle bank
    - How often the bank is emptied
    - Average mass of glass recycled per household