CONTENTS

Please note:

- **The philosophy of your mathematics course is that topics can overlap, so you may encounter Paper 1 material on Paper 2 and vice versa.**
- **The exam questions marked by the symbol  in this book are selected from the following:**
  1. **SEC exam papers (relevant year indicated)**
  2. **Sample exam papers**
  3. **Original and sourced exam-type questions**

Introduction

aims

- ☐ To learn how to revise most effectively
- ☐ To familiarise yourself with the structure of the exam paper
- ☐ To learn how to allocate the correct time for each question
- ☐ To know and understand the words which appear often on the exam paper
- ☐ To familiarise yourself with the syllabus

The aim of this revision book is to help you enhance your grade in your Leaving Certificate. The book is designed to be exam focused. To do this, the book is based not just on the syllabus, but also on the examination paper. Because of this, this revision book can be used in conjunction with **any** textbook.

Throughout this book, **examples and exam-type questions are graded by level of difficulty**.

This level of difficulty is indicated by calculator symbols, as follows: 

The number of calculators shown beside a question helps you know how difficult the question is. One calculator indicates a question which is relatively basic. As the questions get harder, there will be more calculators. Three calculators indicates an average-level question, whereas five calculators indicates that it is a very challenging question. These questions may be beyond some students, but give them a go! **Students hoping to achieve a high grade should aim to complete all of the five calculator questions. The calculator symbol given for each question relates to the most difficult part of that question. Don't be discouraged by a challenging question.** As in the Leaving Certificate exam, difficult questions can sometimes begin with one or two simple parts. You should attempt as much as you can.

It is very important to realise that **you are your own best teacher**. Revision is when you begin to teach yourself. Thus, it is very important for you to start your revision as soon as possible. Make notes while you are revising. If you are having difficulty with a particular question, seek help from your teacher, a friend or a member of your family. As with all subjects, the best examination preparation is to work through past examination or sample papers so that you are familiar with the layout and the style of questions.

Let's start at the beginning. If you want to do well in your Leaving Certificate, then two things are essential:

- Revise effectively.
- Be familiar with the exam paper and so be prepared on the day of the exam.

These may seem obvious, but it's worth taking a moment to think about what these tips mean.

How to revise most effectively

If you are going to do well in the Leaving Certificate, you need to spend quite a bit of time revising. Spending a little time learning how to revise effectively will help you get more from your time and will help you absorb and understand more of the material on the course. Here are some tips to help you revise for maths.

- Find a quiet place where you can work. This place should be dedicated to study and free of potential distractions. Turn off music, the TV, computer and mobile phone.
- Write a study plan. Don't be afraid to ask your parents/teachers/guidance counsellor for help at this stage.
- Do the more challenging revision first, when you are fresh. Trying to focus on difficult problems when you are tired can be counter-productive.
- Maths is based on understanding, so while you can 'learn' some elements of the course, it is important that you develop an understanding of the material.
- Drill and practice are essential ingredients for success in maths.
- Try to link any new material to things you know already. This is learning through association and helps long-term retention.

Study in small chunks of time lasting 25 to 35 minutes. Your memory and concentration will work better if you study in short, frequent bursts.

Don't get hung up on more difficult material. Concentrate on understanding the fundamental concepts and being able to answer all straightforward questions. Then, with time, you can build up to the more challenging problems.

Leaving Certificate examination

Exam focus is critical to exam success. It is important to prepare yourself for the challenges you will face. By learning about the structure of the exam, you will learn how to maximise your points, allocate your time effectively and manage the paper in a calm manner.

The examination paper will be presented in two sections.

Section A – 150 marks
Concepts and Skills

Read the exam paper right through at the start in order to determine which question is the easiest one to start with. Your mind may also be subconsciously processing some of the other problems.

Section B – 150 marks
Contexts and Applications

Start with your best question, then your next best and so on. This way, if you are short of time, at least your best questions will be done.

Time yourself as follows

- Read the paper at the start: 5 minutes
- Section A: 70 minutes
- Section B: 70 minutes
- Review your answers at the end: 5 minutes
- Try to stick closely to these times. If you run out of time on a question, leave it and come back at the end.
- Keep moving through the questions and follow the procedures you have learned.

Attempt marks are valuable, so it is vital that you attempt all questions. Leave **NO** blanks.

Further exam tips

- There is no such thing as rough work in Maths – all work is relevant. If the examiner doesn't know how you reached an answer, even a correct answer, then full marks may not be awarded. Thus, **show all your work**.
- Attempt marks will be awarded for any step in the right direction. Therefore, **make an attempt at each part of the question**. Even if you do not get the correct answer, you can still pick up most of the marks on offer if you show how you worked it out. Also, **draw a diagram where possible** because this can help you see the solution.
- If you cannot finish part of a question, leave a space and come back to it later. **Never scribble out any work or use Tipp-Ex**. Put a single line through it so that the examiner can still read it. In many cases, work that had a line through it received more marks. **Avoid using pencil** because the writing can be very faint and difficult to read.

- It is a good idea to show each stage of a calculation when using a calculator (in case you press a wrong key). Familiarise yourself with your calculator. Know your *booklet of formulae and tables* well and write down any formula that you use.

Your calculator and *booklet of formulae and tables* are two extremely valuable resources to have in the exam. Make sure that you are very familiar with how your calculator works and that you know how to perform all functions on it. Also familiarise yourself with the *booklet of formulae and tables* so that you don't waste any time in the exam trying to find formulae.

Glossary of words used on the examination paper

Write down, state
You can write down your answer without showing any work. However, if you want you can show some workings.

Calculate, find, show that, determine, prove
Obtain your answers by showing all relevant work. Marks are available for showing the steps leading to your final answer or conclusion.

Solve
Find the solution, or root, of an equation. The solution is the value of the variable that makes the left-hand side balance with the right-hand side.

Evaluate
Work out, or find, a numerical value by putting in numbers for letters.

Comment on
After studying the given information or answers, give your opinion on their significance.

Plot
Indicate the position of points on a graph, usually on the x- and y-planes.

Construct
Draw an accurate diagram, usually labelled, using a pencil, ruler, set square, compass and protractor. Leave all constructions on your diagram.

Sketch
Make a rough diagram or graph, labelled if needed.

Hence
You **must** use the answer, or result, from the previous part of the question.

Hence or otherwise
It is recommended that you use the answer, or result, from the previous part of the question, and it is usually best to do this, but other methods are acceptable.

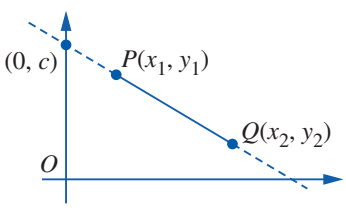
1 Coordinate Geometry of the Line

aims

- To know where to find the coordinate geometry formulae in the *booklet of formulae and tables*
- To learn how to apply these formulae to procedural and in-context examination questions
- To gain the ability, with practice, to recall relevant techniques and tactics for the exam

Coordinate geometry formulae

Nine formulae for coordinate geometry are on pages 18 and 19 of the *booklet of formulae and tables*. Here they are:



$$m = \frac{y_2 - y_1}{x_2 - x_1}$$ slope of PQ

$$|PQ| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$ length of $[PQ]$

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$ midpoint of $[PQ]$

$$y - y_1 = m(x - x_1)$$
$$y = mx + c$$ equation of PQ

$$\frac{1}{2}|x_1y_2 - x_2y_1|$$ area of triangle OPQ

$$\left(\frac{bx_1 + ax_2}{b + a}, \frac{by_1 + ay_2}{a + b}\right)$$ point dividing $[PQ]$ in the ratio $a : b$

$$\frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$$ distance from (x_1, y_1) to the line $ax + by + c = 0$

$$\tan\theta = \pm\frac{m_1 - m_2}{1 + m_1m_2}$$ angles between two lines of slopes m_1 and m_2

In addition, we must also know the following rules:

(i) Parallel lines have equal slopes.

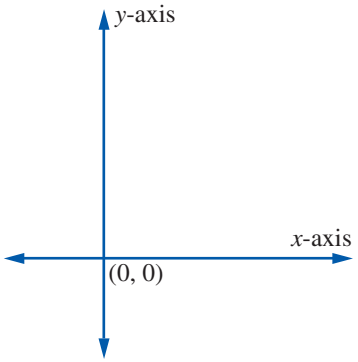
If $l \| k \Leftrightarrow m_l = m_k$

(ii) If two lines are perpendicular then the product of their slopes equals -1.
If $l \perp k \Leftrightarrow (m_l)(m_k) = -1$

(iii) $y = 0$ is the equation of the x-axis.
$x = 0$ is the equation of the y-axis.

key point

m_l is the slope of l and m_k is the slope of k.



exam focus

Rules (i), (ii) and (iii) are not in the *booklet of formulae and tables*. You have to know them!

To verify that a point belongs to a line

Substitute the coordinates of the point into the equation of the line. If the coordinates satisfy the equation, then the point is on the line, otherwise, the point is not on the line.

Example

Is the point $(4, -7)$ on the line $2x - 3y = 27$?
Justify your answer.

Solution

If $(4, -7) \in 2x - 3y = 27$
then $2(4) - 3(-7) = 27$
$8 + 21 \neq 27$
Hence $(4, -7)$ is not on the line $2x - 3y = 27$.

Example

The point $P(k^2, k)$ is on the line $x - 2y = 8$.
Find the coordinates of P where $k \in \mathbb{R}$.

Solution

$(k^2, k) \in x - 2y = 8$
$(k^2) - 2(k) = 8$
$k^2 - 2k - 8 = 0$
Factorising: $(k + 2)(k - 4) = 0$
$k = -2$ or $k = 4$

$P = (k^2, k)$
$k = -2$ then $P = (4, -2)$
$k = 4$ then $P = (16, 4)$

key point

Both points are on the line $x - 2y = 8$

Equation of a line, and finding the point of intersection of two lines

In this section, we revise some straightforward coordinate geometry by tackling an exam question.

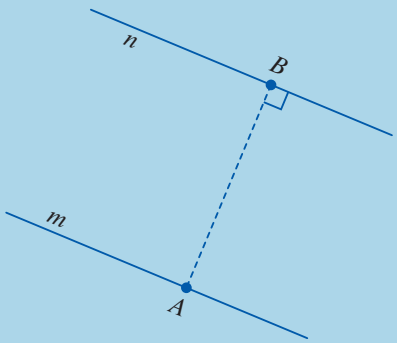
exam Q

(2018 Q.5 (a) and (b))

The line m: $2x + 3y + 1 = 0$ is parallel to the line n: $2x + 3y - 51 = 0$.

(a) Verify that $A(-2, 1)$ is on m.

(b) Find the coordinates of B, the point on the line n closest to A as shown



Solution

(a) Is $A(-2,1) \in 2x + 3y + 1 = 0$

$$2(-2) + 3(1) + 1 = 0?$$
$$-4 + 3 + 1 = 0$$

This is true. Therefore the point A(−2, 1) is on the line.

exam focus

Part (a) was awarded 10 marks.
Part (b) was awarded 10 marks.

(b) Slope of line m: $2x + 3y + 1 = 0$

$$3y = -2x - 1 \text{ has slope } = \frac{-2}{3}$$

Slope of $AB = +\frac{3}{2}$ because AB is perpendicular to line m.

Equation of AB given by $y - y_1 = m(x - x_1)$ $\quad m = \frac{3}{2}$ $\quad (x_1, y_1) = (-2, 1)$

$$y - 1 = \frac{3}{2}(x - (-2))$$
$$2y - 2 = 3x + 6$$
$$-3x + 2y = 8$$

Now find B, the point of intersection of AB and the line n.

$$\begin{array}{r} 2x + 3y = 51 \text{ multiply by } 3 \\ -3x + 2y = 8 \text{ multiply by } 2 \\ \hline 6x + 9y = 153 \\ -6x + 4y = 16 \\ \hline 13y = 169 \\ y = 13 \end{array}$$

Now sub $y = 13$ into (say) $-3x + 2y = 8$

$$\begin{aligned} -3x + 2(13) &= 8 \\ -3x &= 8 - 26 \\ -3x &= -18 \\ x &= 6 \end{aligned}$$

Answer: B = (6, 13)

Transformations of the plane

(a) Translation: A translation moves a point in a straight line.
(b) Central symmetry: Central symmetry is a reflection in a point.
(c) Axial symmetry: Axial symmetry is a reflection in a line.
(d) Axial symmetry in the axes or central symmetry in the origin.

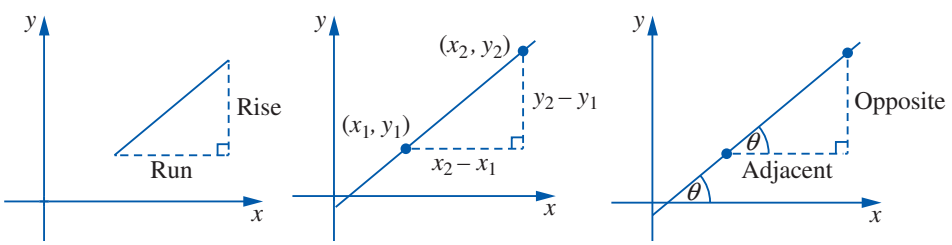
The following three patterns emerge and it is worth memorising them:
1. Axial symmetry in the x-axis $\rightarrow$ change the sign of y
2. Axial symmetry in the y-axis $\rightarrow$ change the sign of x
3. Central symmetry in the origin, (0, 0) $\rightarrow$ change the sign of both x and y

Note: Under a translation or a central symmetry, a line is mapped onto a parallel line.

Slope of a line

Slope of a line, m, given two points. $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

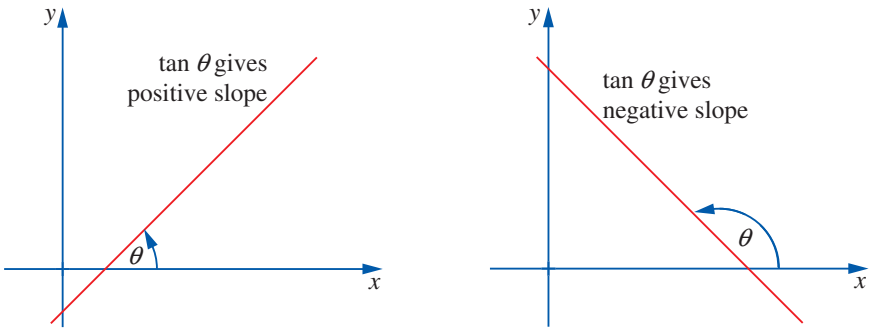
Slope is $\dfrac{\text{rise}}{\text{run}} = \tan\theta$, where θ is the angle the line makes with the positive sense of the x-axis.



We say θ, the angle of inclination, is the angle formed between a line and the positive side of the x-axis.

The angle of inclination is always between 0° and 180°.

- It is always measured anticlockwise from the positive side of the x-axis.
- The slope m of any line is equal to the tangent of its angle of inclination: then $m = \tan\theta$ (where θ = angle of inclination).



The slope of a line when given its equation

To find the slope of a line when given its equation, do the following.

Method 1:

Get y on its own, and the number in front of x is the slope.

Note: The number in front of x is called the **coefficient** of x.
The number on its own is called the y-**intercept**.
In short: write the line in the form
$y = mx + c$.

$y = \underset{\downarrow}{mx} + \underset{\downarrow}{c}$ (see *booklet of formulae and tables*, page 18)

$y =$ (slope)$x +$ (where the line cuts the y-axis)

Method 2:

If the line is in the form $ax + by + c = 0$, then $-\frac{a}{b}$ is the slope.

In other words: Slope $= -\dfrac{\text{Number in front of } x}{\text{Number in front of } y}$

When using this method, make sure every term is on the left-hand side in the given equation of the line.

Lines parallel to the axes

$y = 1$ is a line parallel to the x-axis through the point $(0, 1)$.

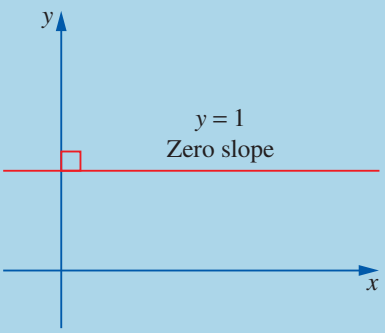
$x = 2$ is a line parallel to the y-axis through the point $(2, 0)$.

key point

$y = 0$ is the equation of the x-axis.
$x = 0$ is the equation of the y-axis.

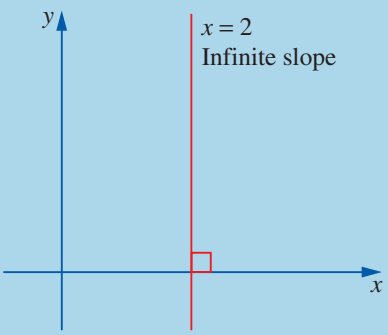
All horizontal lines have an angle of inclination of 0°.

Their slopes are zero.



All vertical lines have an angle of inclination of 90°.

Their slopes are infinitely steep.



exam Q

(2013 Q.3 (a))

The equations of six lines are given:

Line	Equation
h	$x = 3 - y$
i	$2x - 4y = 3$
k	$y = -\frac{1}{4}(2x - 7)$
l	$4x - 2y - 5 = 0$
m	$x + \sqrt{3}y - 10 = 0$
n	$\sqrt{3}x + y - 10 = 0$

Complete the table below by matching each description given to one or more of the lines.

Description	Line(s)
A line with a slope of 2.	
A line which intersects the y-axis at $(0, -2\frac{1}{2})$	
A line which makes equal intercepts on the axes	
A line which makes an angle of 150° with the positive sense of the x-axis	
Two lines which are perpendicular to each other.	

Solution

Use $y = mx + c$ six times

h: $x = 3 - y$

$y = 3 - x$

h has slope $= -1$

i: $2x - 4y = 3$

$2x - 3 = 4y$

$\frac{1}{2}x - \frac{3}{4} = y$

i has a slope $= \frac{1}{2}$

k: $y = -\frac{1}{4}(2x - 7)$

$y = -\frac{1}{2}x + \frac{7}{4}$

k has a slope $= -\frac{1}{2}$

l: $4x - 2y - 5 = 0$

$4x - 5 = 2y$

$2x - \frac{5}{2} = y$

l has a slope $= 2$

Very few candidates answered all five correctly. The marking scheme reflected this.

3 marks for any one correct.

7 marks for any two correct.

9 marks for any four correct.

10 marks for all five correct.

m: $x + \sqrt{3}y - 10 = 0$

$\sqrt{3}y = 10 - x$

$y = \frac{10}{\sqrt{3}} - \frac{1}{\sqrt{3}}x$

m has slope $= -\frac{1}{\sqrt{3}}$

n: $\sqrt{3}x + y - 10 = 0$

$y = 10 - \sqrt{3}x$

m has slope $= -\sqrt{3}$

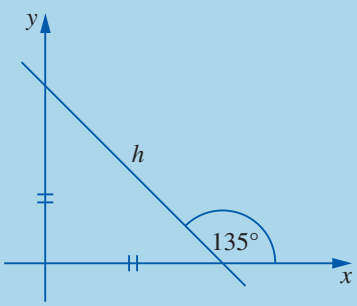
The six slopes above tell us:

- *l has slope* 2
- $k \perp l$ because $\left(-\frac{1}{2}\right)(2) = -1$
- tan 150° = slope $= -\frac{1}{\sqrt{3}}$ slope m
- Equal intercepts $\rightarrow$ slope = tan 135° $= -1$

  Then h has slope $= -1$.
- Finally $\left(0, -2\frac{1}{2}\right) \in l$

  $\left(0, -2\frac{1}{2}\right) \in 4x - 2y - 5 = 0$

  $0 + 5 - 5 = 0$



Answer:

Description	Line(s)
A line with a slope of 2.	l
A line which intersects the y-axis at $(0,-2\frac{1}{2})$	l
A line which makes equal intercepts on the axes	h
A line which makes an angle of 150° with the positive sense of the x-axis	m
Two lines which are perpendicular to each other.	l, k

Example

Investigate whether $A(-4, 3)$, $B(-1, 6)$ and $C(7, 10)$ are collinear.

Collinear: If three or more points lie on the same line, then the points are said to be collinear.

Note: Two points always lie on a line.

Solution

Method 1

The three points either form a straight line or they do not.
To decide which, we will find the slopes of AB and BC.

$$m_{AB} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6-3}{-1-(-4)} = \frac{3}{3} = 1$$

$$m_{BC} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{10-6}{7-(-1)} = \frac{4}{8} = \frac{1}{2}$$

As $m_{AB} \neq m_{BC}$, the points A, B and C are not collinear.
Note: We could have found the slope of m_{AC} as one of the two slopes.

Method 2

To find the area of $\triangle ABC$, use translation $(-4, 3) \rightarrow (0, 0)$ to get:

$A = (-4, 3) \rightarrow (0, 0)$ (Add 4 to x, subtract 3 from y)

$B = (-1, 6) \rightarrow (3, 3) = (x_1, y_1)$

$C = (7, 10) \rightarrow (11, 7) = (x_2, y_2)$

$$\therefore \text{ Area } \triangle ABC = \frac{1}{2}|x_1y_2 - x_2y_1| \quad \text{(see } \textit{booklet of formulae and tables}\text{, page 18)}$$
$$= \frac{1}{2}|(3)(7) - (11)(3)|$$
$$= \frac{1}{2}|21 - 33| = \frac{1}{2}|-12| = 6$$

Since the area of $\triangle ABC \neq 0$, the three points A, B and C do not form a straight line $\Rightarrow$ A, B and C are not collinear.

If the area of $\triangle PQR = 0$, then we can state P, Q, R are collinear.

Division of a line segment in a given ratio

The coordinates of the point $C(x, y)$ which divides the line segment $P(x_1, y_1)$ and $Q(x_2, y_2)$ internally in the ratio $a : b$ is given by:

Internal divisor

$$C(x, y) = \left(\frac{bx_1 + ax_2}{b + a}, \frac{by_1 + ay_2}{b + a}\right)$$

(see *booklet of formulae and tables*, page 18)

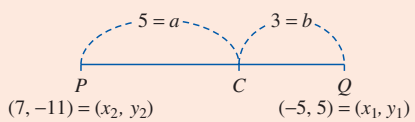
Example

(i) $P(7, -11)$ and $Q(-5, 5)$ are two points. C is a point on $[PQ]$ such that $|PC| : |CQ| = 5 : 3$. Find the coordinates of C.

(ii) The point $R\left(-\frac{1}{2}, -1\right)$ divides the line segment $|VW|$ such that $|VR| : |RW| = 1 : 4$. If the coordinates of V are $(3, -3)$, find the coordinates of W.

Solution

(i) $$C = \left(\frac{(5)(-5) + (3)(7)}{5 + 3}, \frac{(5)(5) + (3)(-11)}{5 + 3}\right)$$
$$C = \left(\frac{-25 + 21}{8}, \frac{25 - 33}{8}\right) = \left(-\frac{1}{2}, -1\right)$$



(ii) Using a translation