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Please note:

- The philosophy of your mathematics course is that topics can overlap, so you may encounter Paper 1 material on Paper 2 and vice versa.
- The exam questions marked by the symbol in this book are selected from the following:
 SEC exam papers (relevant year indicated)
 - 2. Sample exam papers
 - 3. Original and sourced exam-type questions

Introduction

To learn how to revise most effectively

aims

- □ To familiarise yourself with the structure of the exam paper
- To learn to allocate the correct time for each question
- To know and understand the words which appear often on the exam paper
- □ To familiarise yourself with the syllabus

The aim of this revision book is to help you enhance your grade in your Leaving Certificate. This book is designed to be exam focused. To do this, the book is based not just on the syllabus, but also on the examination paper. As a result, this revision book can be used in conjunction with **any** textbook.

Throughout this book, examples and exam-type questions are graded by level of difficulty.

The level of difficulty is indicated by calculator symbols, as follows:

The number of calculators shown beside a question indicates how difficult the question is. One calculator indicates a question which is relatively basic. As the questions get harder, the symbol will have more calculators. Three calculators indicates an average-level question, whereas five calculators indicates that it is a very challenging question. These questions may be beyond some students, but give them a go! **Students hoping to achieve a high grade should aim to complete all of the 'five calculator' questions**. The calculator symbol given for each question relates to the most difficult part of that question. **Do not be discouraged by a challenging question**. As in the Leaving Certificate exam, difficult questions can sometimes begin with one or two simple parts. You should attempt as much as you can.

It is very important to realise that **you are your own best teacher**. Revision is when you begin to teach yourself. Thus, it is very important for you to start your revision as soon as possible. Make notes while you are revising. If you are having difficulty with a particular question, seek help from your teacher, a friend or a member of your family. As with all subjects, the best examination preparation is to work through past examination or sample papers so that you are familiar with the layout and style of questions.

So let's start at the beginning. If you want to do well in your Leaving Certificate, then two things are essential:

- Revise effectively.
- Be familiar with the exam paper and so be prepared on the day of the exam.

These may seem obvious, but it's worth taking a moment to think about what these tips mean.

How to revise most effectively

If you are going to do well in the Leaving Certificate, you are going to spend quite a bit of time revising. Spending a little time learning how to revise effectively will help you get more from your time and help you absorb and understand more of the material on the course. Here are some tips to help you revise for maths.

- Find a quiet place where you can work. This place should be dedicated to study and free of potential distractions. Turn off music, the TV, computer and mobile phone.
- Write a study plan. Don't be afraid to ask your parents/teachers/guidance counsellor for help at this stage.
- Do the more challenging revision first, when you are fresh. Trying to focus on difficult problems when you are tired can be counterproductive.



Study in small chunks lasting 25 to 35 minutes. Your memory and concentration will work better if you study in short, frequent bursts.

- Maths is based on understanding, so while you can 'learn' some elements of the course, it is important that you develop an understanding of the material.
- Drill and practice are essential ingredients for success in maths.
- Try to link any new material to things you know already. This is learning through association and helps long-term retention.



Don't get hung up on more difficult material. Concentrate on understanding the fundamental concepts and being able to answer all of the straightforward questions. Then, with time, you can build up to the more challenging problems.

Leaving Certificate examination

Exam focus is critical to exam success. It is important to prepare yourself for the challenge you will face. By learning about the structure of the exam, you will learn how to maximise your points, allocate your time effectively and manage the paper without panic.

The order of the questions is not set and some questions may include cross-syllabus topics. The examination paper will be presented in two sections, as follows:

Section A – 150 marks

Concepts and Skills



Read the exam paper right through at the start in order to determine which question is the easiest one to start with. Your mind may also be subconsciously processing some of the other problems.

Section B – 150 marks

Contexts and Applications



Start with your best question, then your next best and so on. This way, if you are short of time, at least your best questions will be done.

Time yourself as follows

- Reading the paper at the start: 5 minutes
- Section A : 70 minutes
- Section B : 70 minutes
- Reviewing your answers at the end: 5 minutes
- Try to stick closely to these times. If you run out of time on a question, leave it and come back at the end.



Rule of thumb for timing yourself during the exam:

Time spent on question = $\frac{1}{2}$ (marks for question)

That is, a 25-mark question should take no more than 12.5 minutes.

Further exam tips

- There is no such thing as rough work in maths all work is relevant. If the examiner doesn't know how you reached an answer, even a correct answer, then full marks will not usually be awarded. Thus, **show all your work**.
- Attempt marks will be awarded for any step in the right direction. Therefore, **make an attempt at each part of the question**. Even if you do not



Attempt marks (partial credit) are valuable, so it is vital that you attempt all questions. Leave **NO** blanks.

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get the correct answer, you can still pick up most of the marks on offer if you show how you worked it out. Also, **draw a diagram where possible**, because this can help in seeing the solution.

- If you cannot finish part of a question, leave a space and come back to it later. Never scribble out any work or use Tipp-Ex. Put a single line through it so that the examiner can still read it. In many cases, work that had a line through it received more marks. Avoid using pencil because the writing can be very faint and difficult to read.
- It is a good idea to show each stage of a calculation when using a calculator (in case you press a wrong key). Familiarise yourself with your calculator. Know your *booklet of formulae and tables* well and write down any formula that you use.



Your calculator and *booklet of formulae and tables* are two extremely valuable resources to have in the exam. Make sure that you are very familiar with how your calculator works and that you know how to perform all functions on it. Familiarise yourself with the *booklet of formulae and tables* so that you don't waste any time in the exam trying to find formulae.

Glossary of words used on the examination paper

Write down, state

You can write down your answer without showing any work. However, if you want you can show some workings.

Calculate, find, show that, determine, prove

Obtain your answers by showing all relevant work. Marks are available for showing the steps leading to your final answer or conclusion.

Solve

Find the solution, or root, of an equation. The solution is the value of the variable that makes the left-hand side balance with the right-hand side.

Evaluate

Usually to work out, or find, a numerical value by putting in numbers for letters.

Comment on

After studying the given information or your answers, give your opinion on their significance.

Plot

Indicate the position of points on a graph, usually on the *x*- and *y*-planes.

Construct

Draw an accurate diagram, usually labelled, using a pencil, ruler, set square, compass and protractor. Leave all constructions on your diagram.

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Sketch

Make a rough diagram or graph, labelled if needed.

Hence

You *must* use the answer or result from the previous part of the question.

Hence or otherwise

It is recommended that you use the answer or result from the previous part of the question, but other methods are acceptable.

Syllabus and checklist for Leaving Certificate Higher Level Maths Paper 1 exam

The philosophy of your mathematics course is that topics can overlap, so you may encounter Paper 2 material on Paper 1 and vice versa.

Throughout your course you will be asked to apply your knowledge and skills to solve problems in familiar and unfamiliar contexts. In problem solving, you should use some of the following strategies:

- Trial and improvement
- Draw a diagram
- Look for a pattern
- Act it out
- Draw a table

- Simplify the problem
- Use an equation
- Work backwards
- Eliminate possibilities

The syllabus stresses that in all aspects of the Leaving Certificate Maths course, students should be able to:

- Explore patterns and formulate conjectures
- Explain findings
- Justify conclusions
- Communicate mathematics verbally and in written form
- Apply their knowledge and skills to solve problems in familiar and unfamiliar contexts
- Analyse information presented verbally and translate it into mathematical form
- Devise, select and use appropriate mathematical models, formulae or techniques to process information and to draw relevant conclusions

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Factorising and simplifying expressions

You must be able to factorise expressions using the following methods:

Take out common terms	Factorise by grouping	
ab + ad = a(b + d)	ab + ad + cb + cd = (a + c)(b + d)	
Factorise a trinomial	Difference of two squares	
$a^2 - 2ab + b^2 = (a - b)(a - b)$	$a^2 - b^2 = (a + b)(a - b)$	
Difference of two cubes	Sum of two cubes	
$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$	$a^{3} + b^{3} = (a + b)(a^{2} - ab + b^{2})$	

Factorising is a basic and vital skill for you to have throughout your maths course. You must be able to factorise expressions quickly and easily. This will take practice, but it is worthwhile spending time on.

Example

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Factorise the following:

(i) $3p^2 + 6pq$ (ii) $6ab + 12bc - 8ac - 9b^2$ (iii) $3x^2 - 12y^2$

Solution

(i)
$$3p^2 + 6pq$$

 $3p(p + 2q)$
(ii) $6ab + 12bc - 8ac - 9b^2$
 $6ab - 9b^2 - 8ac + 12bc$
 $3b(2a - 3b) - 4c(2a - 3b)$
(iii) $3x^2 - 12y^2$
 $3(x^2 - 4y^2)$
 $3(x + 2y)(x - 2y)$

4(a + 2b)(a² - (a)(2b) + (2b)²)4(a + 2b)(a² - 2ab + 4b²)

Example

Factorise the following:

(i) $2x^2 - 7x - 15$ (ii) $64 - 27x^3$ (iii) $4a^3 + 32b^3$

Solution

(i) $2x^2 - 7x - 15$ (2x + 3)(x - 5)

(iii) $4a^3 + 32b^3$ $4(a^3 + 8b^3)$ $4((a)^3 + (2b)^3)$ (ii) $64 - 27x^3$ $(4)^3 - (3x)^3$ $(4 - 3x)(4^2 + (4)(3x) + (3x)^2)$ $(4 - 3x)(16 + 12x + 9x^2)$

Example Simplify $\frac{x^3 + 7x^2 + 12x}{x^2 + 2x - 3}$. Solution Factorise the top and bottom: $\frac{x(x^2 + 7x + 12)}{x^2 + 2x - 3}$ $\frac{x(x + 3)(x + 4)}{(x + 3)(x - 1)}$ (divide top and bottom by (x + 3)) $\frac{x(x + 4)}{x - 1}$ (in simplest form)



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$\frac{5(x) - 3(1) - 1(2x - 3)}{x(2x - 3)}$	(common denominator $x(2x - 3)$)
$\frac{5x - 3 - 2x + 3}{x(2x - 3)}$	
$\frac{3x}{x(2x-3)}$	(divide top and bottom by <i>x</i>)
$\frac{3}{2x-3}$	





Solution $f(x) \div g(x) = \frac{x^3 - 1}{x^2 - 1} \div \frac{x^2 + x + 1}{x^2 - x - 2}$		
$=\frac{x^3-1}{x^2-1}\times\frac{x^2-x-2}{x^2+x+1}$	(invert the second fraction and multiply)	
$=\frac{(x-1)(x^2+x+1)}{(x+1)(x-1)}\times\frac{(x-2)(x+1)}{x^2+x+1}$	(factorise each part)	
$=\frac{1}{1}\times\frac{(x-2)}{1}$	(divide top and bottom by $(x - 1)(x^2 + x + 1)(x + 1))$	
= x - 2, which is in the form $ax + b$		
Therefore $a = 1, b = -2$.		

Changing the subject of a formula

When we rearrange a formula so that one of the variables is given in terms of the others, we are said to be **changing the subject of the formula or manipulating the formula**. The rules in changing the subject of a formula are the same as when solving an equation. That is, we can:

- 1. Add or subtract the same quantity to both sides.
- 2. Multiply or divide both sides by the same quantity.
- 3. Square both sides, cube both sides, etc.
- 4. Take the square root of both sides, take the cube root of both sides, etc.

Note: Whatever letter comes after the word 'express' is to be on its own.



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Example If $c = \frac{b^2 - ac}{b + a}$, express *a* in terms of the other variables. Solution $c = \frac{b^2 - ac}{b + a}$ $(b + a)c = (b + a)\left(\frac{b^2 - ac}{b + a}\right)$ (multiply both sides by (b + a)) $(b+a)c = (b^2 - ac)$ $bc + ac = b^2 - ac$ (multiply out brackets) $ac = b^2 - ac - bc$ (subtract *bc* from both sides) $ac + ac = b^2 - bc$ (add *ac* to both sides) $2ac = b^2 - bc$ $\frac{2ac}{2c} = \frac{b^2 - bc}{2c}$ (divide both sides by 2c) $a = \frac{b^2 - bc}{2c}$

Example

(i) If $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$, express v in terms of the other variables.

(ii) Hence, determine the value of v when f = 15 and u = 20.

Solution

(i)
$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$
$$fuv\left(\frac{1}{f}\right) = fuv\left(\frac{1}{u}\right) + fuv\left(\frac{1}{v}\right) \quad (\text{multiply all parts by } fuv)$$
$$uv = fv + fu$$
$$uv - fv = fv + fu - fv \qquad (\text{subtract } fv \text{ from both sides})$$
$$uv - fv = fu$$
$$v(u - f) = fu \qquad (\text{factorise out } v)$$
$$\frac{v(u - f)}{(u - f)} = \frac{fu}{(u - f)} \qquad (\text{divide both sides by } (u - f))$$
$$v = \frac{fu}{u - f}$$

(ii)
$$f = 15$$
 and $u = 20$: $v = \frac{fu}{u - f}$
 $v = \frac{(15)(20)}{20 - 15} = \frac{300}{5} = 60$

The time taken, in seconds, for a satellite to complete an orbit of the Earth is given by the formula:

$$T = \sqrt{\frac{4\pi^2 r^3}{GM}}$$

where r = radius of rotation from the centre of the Earth

G = universal gravitational constant

M = mass of the Earth.

- (i) Express the radius of rotation, *r*, in terms of the other variables.
- (ii) The International Space Station (ISS) orbits the Earth once every 91 minutes. Given that the value for $G = 6.67 \times 10^{-11}$ and the mass of the Earth is 6.4×10^{24} , find the radius of rotation of the ISS, correct to the nearest metre.
- (iii) Find the height the ISS is above the surface of the Earth, given that the radius of the Earth is 6,371 km. Give your answer to the nearest kilometre.

Solution

 $4\pi^2$

(i)

$$T = \sqrt{\frac{4\pi^2 r^3}{GM}}$$

$$T^2 = \left(\sqrt{\frac{4\pi^2 r^3}{GM}}\right)^2 \text{ (square both sides)}$$

$$T^2 = \frac{4\pi^2 r^3}{GM}$$

$$GM(T^2) = GM\left(\frac{4\pi^2 r^3}{GM}\right) \text{ (multiply both sides by } GM\text{)}$$

$$GMT^2 = 4\pi^2 r^3$$

$$\frac{GMT^2}{4\pi^2} = r^3 \text{ (divide both sides by } 4\pi^2\text{)}$$

$$3\sqrt{\frac{GMT^2}{2\pi^2}} = r \text{ (take cube root of both sides)}$$

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Undetermined coefficients

When two expressions in x (or any other variable) are equal to one another for all values of x, we can equate the coefficients of the same powers of x in the two expressions. This is known as the **principle of undetermined coefficients**.

Method:

- 1. Remove all fractions and brackets.
- 2. Form equations by equating coefficients of like terms.
- 3. Solve the equations to find the coefficients.

