CONTENTS

Please note:

- The philosophy of your mathematics course is that topics can overlap, so you may encounter Paper 1 material on Paper 2 and vice versa.
- The exam questions marked by the symbol 

in this book are selected from the following:
  1. SEC exam papers (relevant year indicated)
  2. Sample exam papers
  3. Original and sourced exam-type questions

Introduction

aims

- ☐ To learn how to revise most effectively
- ☐ To familiarise yourself with the structure of the exam paper
- ☐ To learn to allocate the correct time for each question
- ☐ To know and understand the words which appear often on the exam paper
- ☐ To familiarise yourself with the syllabus

The aim of this revision book is to help you enhance your grade in your Leaving Certificate. The book is designed to be exam-focused. To do this, the book is based not just on the syllabus, but also on the examination paper. Because of this, this revision book can be used in conjunction with **any** textbook.

Throughout this book, **examples and exam-type questions are graded by level of difficulty**.

This level of difficulty is indicated by calculator symbols, as follows: 

The number of calculators shown beside a question helps you know how difficult the question is. One calculator indicates a question which is relatively basic. As the questions get harder, the symbol will have more calculators. Three calculators indicates an average-level question, whereas five calculators indicates that it is a very challenging question. These questions may be beyond some students, but give them a go! **Students hoping to achieve a high grade should aim to complete all of the 'five calculator' questions.** The calculator symbol given for each question relates to the most difficult part of that question. **Don't be discouraged by a challenging question.** As in the Leaving Certificate exam, difficult questions can sometimes begin with one or two simple parts. You should attempt as much as you can.

It is very important to realise that **you are your own best teacher**. Revision is when you begin to teach yourself. Thus, it is very important for you to start your revision as soon as possible. Make notes while you are revising. If you are having difficulty with a particular question, seek help from your teacher, a friend or a member of your family. As with all subjects, the best examination preparation is to work through past examination or sample papers so that you are familiar with the layout and style of questions.

So let's start at the beginning. If you want to do well in your Leaving Certificate, then two things are essential:

- Revise effectively.
- Be familiar with the exam paper and so be prepared on the day of the exam.

These may seem obvious, but it's worth taking a moment to think about what these tips mean.

How to revise most effectively

If you are going to do well in the Leaving Certificate, you are going to spend quite a bit of time revising. Spending a little time learning how to revise effectively will help you get more from your time and will help you absorb and understand more of the material on the course. Here are some tips to help you revise for maths.

- Find a quiet place where you can work. This place should be dedicated to study and free of potential distractions. Turn off music, the TV, computer and mobile phone.
- Write a study plan. Don't be afraid to ask your parents/teachers/guidance counsellor for help at this stage.
- Do the more challenging revision first, when you are fresh. Trying to focus on difficult problems when you are tired can be counter-productive.
- Maths is based on understanding, so while you can 'learn' some elements of the course, it is important that you develop an understanding of the material.
- Drill and practice are essential ingredients for success in maths.
- Try to link any new material to things you know already. This is learning through association and helps long-term retention.

Study in small chunks of time lasting 25 to 35 minutes. Your memory and concentration will work better if you study in short frequent bursts.

Don't get hung up on more difficult material. Concentrate on understanding the fundamental concepts and being able to answer all straightforward questions. Then, with time, you can build up to the more challenging problems.

Leaving Certificate examination

Exam focus is critical to exam success. It is important to prepare yourself for the challenges you will face. By learning about the structure of the exam, you will learn how to maximise your points, allocate your time effectively and manage the paper without panic.

The order of the questions is not set and some questions may include cross-syllabus topics. The examination paper will be presented in two sections, as follows:

Section A – 150 marks
Concepts and Skills

Section B – 150 marks
Contexts and Applications

Read the exam paper right through at the start in order to determine which question is the easiest one to start with. Your mind may also be subconsciously processing some of the other problems.

Time yourself as follows

- Read the paper at the start: 5 minutes.
- Section A: 70 minutes.
- Section B: 70 minutes.
- Review your answers at the end: 5 minutes.
- Try to stick closely to these times. If you run out of time on a question, leave it and come back at the end.

Start with your best question, then your next best and so on. This way, if you are short of time, at least your best questions will be done.

Rule of thumb for timing yourself during the exam:

Time spent on question = ½ (marks for question)

That is, a 25-mark question should take no more than 12·5 minutes.

Further exam tips

- There is no such thing as rough work in Maths – all work is relevant. If the examiner doesn't know how you reached an answer, even a correct answer, then full marks may not be awarded. Thus, **show all your work**.
- Attempt marks (partial credit) will be awarded for any step in the right direction. Therefore, **make an attempt at each part of the question**. Even if you do not get the correct answer, you can still pick up most of the marks on offer if you show how you worked it out. Also, **draw a diagram where possible** because this can help you see the solution.

Attempt marks (partial credit) are valuable, so it is vital that you attempt all questions.

Leave **NO** blanks.

- If you cannot finish part of a question, leave a space and come back to it later. **Never scribble out any work or use Tipp-Ex.** Put a single line through it so that the examiner can still read it. In many cases, work that had a line through it received more marks. **Avoid using pencil** because the writing can be very faint and difficult to read.
- It is a good idea to show each stage of a calculation when using a calculator (in case you press a wrong key). Familiarise yourself with your calculator. Know your *booklet of formulae and tables* well and write down any formulae that you use.

Your calculator and *booklet of formulae and tables* are two extremely valuable resources to have in the exam. Make sure that you are very familiar with how your calculator works and that you know how to perform all functions on it. Also familiarise yourself with the *booklet of formulae and tables* so that you don't waste any time in the exam trying to find formulae.

Glossary of words used on the examination paper

Write down, state
You can write down your answer without showing any work. However, you can show some workings if you want to.

Calculate, find, show that, determine, prove
Obtain your answers by showing all relevant work. Marks are available for showing the steps leading to your final answer or conclusion.

Solve
Find the solution, or root, of an equation. The solution is the value of the variable that makes the left-hand side balance with the right-hand side.

Evaluate
Work out, or find, a numerical value by putting in numbers for letters.

Comment on
After studying the given information or answers, give your opinion on their significance.

Plot
Indicate the position of points on a graph, usually on the x- and y-planes.

Construct
Draw an accurate diagram, usually labelled, using a pencil, ruler, set square, compass and protractor. Leave all constructions on your diagram.

Sketch
Make a rough diagram or graph, labelled if needed.

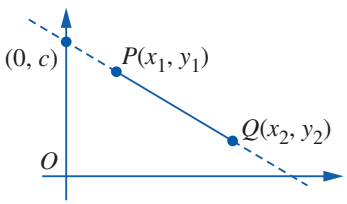
1 Coordinate Geometry of the Line

aims

- To know where to find the coordinate geometry formulae in the *booklet of formulae and tables*
- To learn how to apply these formulae to procedural and in-context examination questions
- To gain the ability, with practice, to recall and select the appropriate technique required by the exam questions

Coordinate geometry formulae

Six formulae for coordinate geometry are on page 18 of the *booklet of formulae and tables*. Here they are:



$$m = \frac{y_2 - y_1}{x_2 - x_1}$$ slope of PQ

$$|PQ| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$ length of $[PQ]$

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$ midpoint of $[PQ]$

$$y - y_1 = m(x - x_1)$$
$$y = mx + c$$ equation of PQ

$$\frac{1}{2}|x_1y_2 - x_2y_1|$$ area of triangle OPQ

In addition, we must also know the following rules:

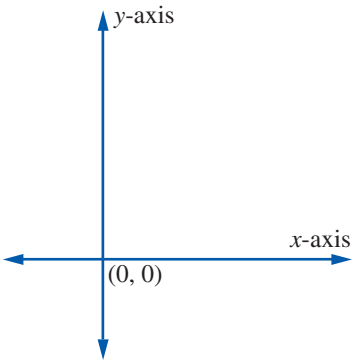
(i) Parallel lines have equal slopes.
If $l \| k \Leftrightarrow m_l = m_k$

(ii) If two lines are perpendicular then the product of their slopes equals -1.
If $l \perp k \Leftrightarrow (m_l)(m_k) = -1$

key point

m_l is the slope of l and m_k is the slope of k.

(iii) $y = 0$ is the equation of the x-axis.
$x = 0$ is the equation of the y-axis.



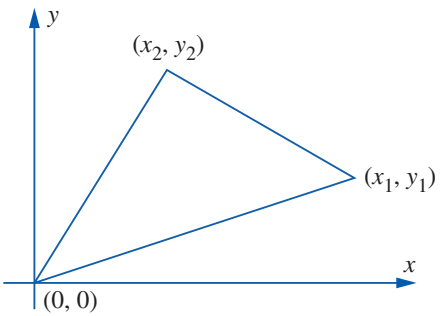
exam focus

Rules (i), (ii) and (iii) are not in the *booklet of formulae and tables*. You have to know them!

Area of a triangle

The area of a triangle with vertices (corners) $(0, 0)$, (x_1, y_1) and (x_2, y_2) is given by the formula:

Area of triangle $= \frac{1}{2}|x_1y_2 - x_2y_1|$ (see *booklet of formulae and tables*, page 18)



key point

1. Always decide which point is (x_1, y_1) and which point is (x_2, y_2) before you use the formula.
2. The modulus symbol, $||$, is included to make sure your answer is positive. This is because you cannot have a negative area. Therefore, if the above formula gives a negative answer, simply ignore the negative sign, e.g. $\frac{1}{2}|-10| = \frac{1}{2}(10) = 5$.
3. If none of the vertices is at the origin, simply select one of the vertices and map (move) it to the point (0, 0) by a translation. Then apply the same translation to the other two vertices to get (x_1, y_1) and (x_2, y_2).

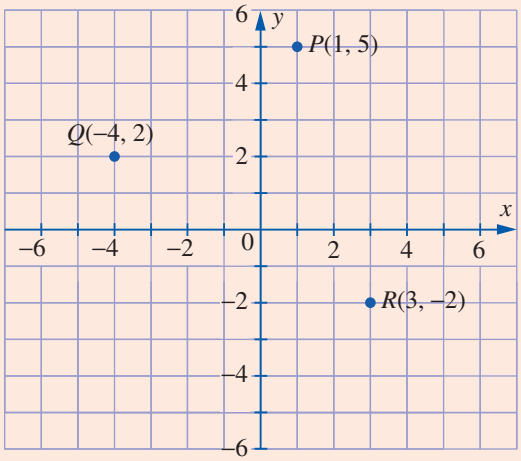
Example

(i) Plot the points $P(1, 5)$, $Q(-4, 2)$ and $R(3, -2)$ on a graph.

(ii) Find the area of the triangle PQR.

Solution

(i)



(ii) Map (move) the point $(1, 5)$ to $(0, 0)$.

Rule: Subtract 1 from x, subtract 5 from y.

$P(1, 5)$	$Q(-4, 2)$	$R(3, -2)$
↓	↓	↓
$(0, 0)$	$(-5, -3)$	$(2, -7)$
	(x_1, y_1)	(x_2, y_2)

$x_1 = -5, \quad y_1 = -3, \quad x_2 = 2, \quad y_2 = -7$

Area of triangle

$$= \tfrac{1}{2}|x_1y_2 - x_2y_1|$$
$$= \tfrac{1}{2}|(-5)(-7) - (2)(-3)|$$
$$= \tfrac{1}{2}|35 + 6|$$
$$= \tfrac{1}{2}|41|$$
$$= 20\tfrac{1}{2} \text{ square units}$$

To find the area of a quadrilateral (four-sided figure), divide it into two triangles.

If the quadrilateral is a **parallelogram**, then the areas of both triangles are equal. Therefore, all that is needed is to find the area of one triangle and double it.

Intersecting lines

To find the point where two lines intersect, we solve the equations of the lines using the method of simultaneous equations, from algebra.

Simultaneous linear equations in two variables are solved with the following steps.

1. Write both equations in the form $ax + by = k$ and label the equations ① and ②.
2. Multiply one or both of the equations by a number in order to make the coefficients of x or y the same, but of opposite sign.
3. Add to remove the variable with equal coefficients but of opposite sign.
4. Solve the resultant equation to find the value of the remaining unknown (x or y).
5. Substitute this value in equation ① or ② to find the value of the other unknown.

Junior Cycle revision example

When geese fly in formation, they form an inverted V shape.

(i) If the lines of geese can be represented by the equations $2x + y - 11 = 0$ and $3x - 2y - 6 = 0$, find the coordinates of the leading goose.

(ii) After 1 hour, the leading goose has flown to a point $(37, 67)$.
Assuming the geese flew in a straight line and taking each unit to represent 1 km, find the distance travelled by the geese to the nearest km.

(iii) Hence, find the average flying speed in m/s.

Solution

(i) Solving the linear equations in two variables:

$$\begin{array}{rll} 2x + y = 11 & & ① \\ 3x - 2y = 6 & & ② \\ \hline 4x + 2y = 22 & & ① \times 2 \\ 3x - 2y = 6 & & ② \\ \hline 7x = 28 & \text{(add)} & \\ x = 4 & & \end{array}$$

Put $x = 4$ into ① or ②.

$$2x + y = 11 \quad ①$$

$$2(4) + y = 11$$
$$8 + y = 11$$
$$y = 3$$

$\therefore$ The leading goose is at $(4, 3)$.

(ii) Use distance formula $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ (see *booklet of formulae and tables*, page 18)

Let $(x_1, y_1) = (4, 3)$ and $(x_2, y_2) = (37, 67)$

$$\text{Distance} = \sqrt{(37 - 4)^2 + (67 - 3)^2} = \sqrt{1{,}089 + 4{,}096} = \sqrt{5{,}185}$$
$$= 72{\cdot}00694411$$

Distance to nearest km $= 72$ km

(iii) $\text{Speed} = \dfrac{\text{Distance}}{\text{Time}} = \dfrac{72 \times 1{,}000}{60 \times 60} = 20$ m/sec

The exam may contain in-context questions at any stage. Be prepared to employ techniques learned elsewhere, as in the above question, where $\text{Speed} = \dfrac{\text{Distance}}{\text{Time}}$, which seems to have no link to coordinate geometry, yet it was required in this question.

Example

$A(8, 5)$ and $B(-10, 11)$ are two points. Find the midpoint of $[AB]$.

Solution

$$\text{Midpoint formula} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$ (see *booklet of formulae and tables*, page 18)

Let $(x_1, y_1) = (8, 5)$ and $(x_2, y_2) = (-10, 11)$

$$\text{Midpoint} = \left(\frac{8 - 10}{2}, \frac{5 + 11}{2}\right) = \left(\frac{-2}{2}, \frac{16}{2}\right) = (-1, 8)$$

In some questions we will be given the midpoint and one end point of a line segment and be asked to find the other end point.

To find the other end point, use the following method:

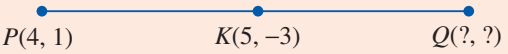
1. Make a rough diagram.
2. Find the translation that maps (moves) the given end point to the midpoint.
3. Apply the same translation to the midpoint to find the other end point.

Example

If $K(5, -3)$ is the midpoint of $[PQ]$ and $P = (4, 1)$, find the coordinates of Q.

Solution

1. Rough diagram



2. Translation from P to K, $\overrightarrow{PK}$. Rule: Add 1 to x, take 4 from y.
3. Apply this translation to K:

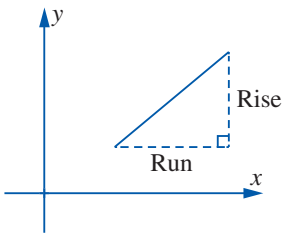
$$K(5, -3) \longrightarrow (5 + 1, -3 - 4) = (6, -7)$$

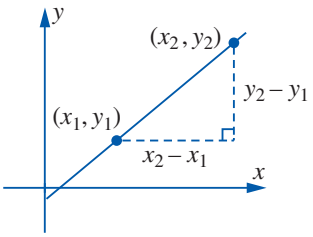
$\therefore$ The coordinates of Q are $(6, -7)$.

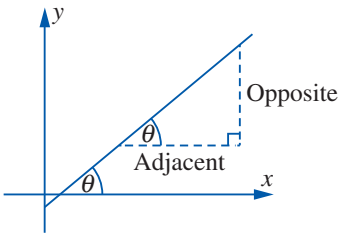
Slope of a line

Slope of a line, m, given two points: $m = \dfrac{y_2 - y_1}{x_2 - x_1}$ (see *booklet of formulae and tables*, page 18)

Slope is $\dfrac{\text{Rise}}{\text{Run}} = \tan\theta$, where θ is the angle the line makes with the positive sense of the x-axis.





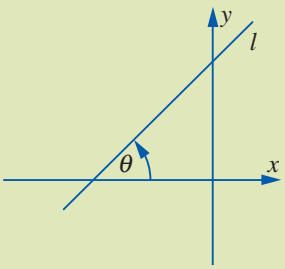


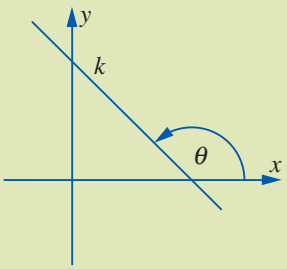
key point

We say θ, the angle of inclination, is the angle formed between a line and the positive side of the x-axis.

The angle of inclination is always between 0° and 180°.

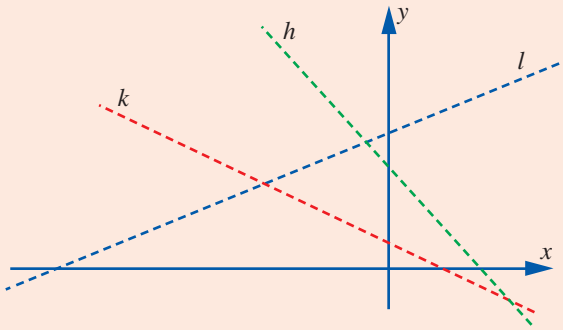
- It is always measured anticlockwise from the positive side of the x-axis.





- The line l has a positive slope where $0° < \theta < 90°$. (acute angle)
- The line k has a negative slope where $90° < \theta < 180°$. (obtuse angle)
- When $\theta = 0°$, the slope of the line is 0. This would be a horizontal line.
- When $\theta = 90°$, the slope of the line is not defined. This would be a vertical line.

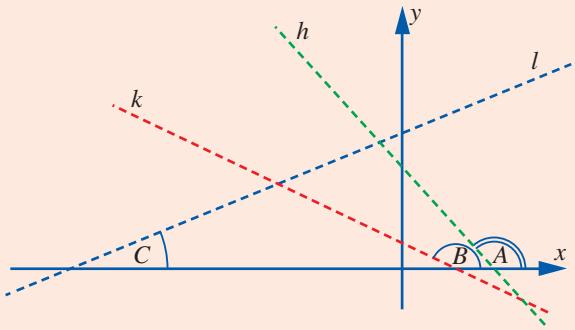
Example



Which of the above lines have a positive slope? Justify your answer.

Solution

Mark the angles of inclination A, B and C, as in the diagram.



By observation, angle C is the only angle less than 90°.

$\therefore$ tan $\angle C$ gives the only positive slope.

Since line l is associated with angle C, we conclude that line l has a positive slope.

Some exam solutions may be short but wordy. Alternatively, since l is the only line going up (reading the graph from left to right), the slope of l is positive.