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Please note:

- The philosophy of your mathematics course is that topics can overlap, so you may encounter Paper 1 material on Paper 2 and vice versa.
- The exam questions marked by the symbol  in this book are selected from the following:
 1. SEC exam papers (relevant year indicated)
 2. Sample exam papers
 3. Original and sourced exam-type questions

Introduction

aims

- To learn how to revise most effectively
- To familiarise yourself with the structure of the exam paper
- To learn how to allocate the correct time for each question
- To know and understand the words which appear often on the exam paper
- To familiarise yourself with the syllabus

The aim of this revision book is to help you get as high a mark as possible in your Leaving Certificate. This book is designed to be exam-focused. To do this, the book is based not just on the syllabus, but also on the examination paper. As a result, this revision book can be used in conjunction with **any** textbook.

Throughout this book, **examples and exam-type questions are graded by level of difficulty.**

This level of difficulty is indicated by calculator symbols, as follows: 

The number of calculators shown beside a question helps you know how difficult the question is. One calculator indicates a question which is relatively basic. As the questions get harder the symbol will have more calculators. Three calculators indicates an average, level question, whereas five calculators indicates that it is a very challenging question. These questions may be beyond some students, but give them a go! **Students hoping to achieve a high grade should aim to complete all of the five calculator questions.** The calculator symbol given for each question relates to the most difficult part of that question. **Do not be discouraged by a challenging question.** As in the Leaving Certificate exam, difficult questions can sometimes begin with one or two simple parts. You should attempt as much as you can.

It is very important to realise that **you are your own best teacher.** Revision is when you begin to teach yourself. Thus, it is very important for you to start your revision as soon as possible. Make notes while you are revising. If you are having difficulty with a particular question, seek help from your teacher, a friend or a member of your family. As with all subjects, the best examination preparation is to work through past examination or sample papers so that you are familiar with the layout and style of questions.

Let's start at the beginning. If you want to do well in your Leaving Certificate, then two things are essential:

- Revise effectively.
- Be familiar with the exam paper and so be prepared on the day of the exam.

These may seem obvious, but it's worth taking a moment to think about what these tips mean.

How to revise most effectively

If you are going to do well in the Leaving Certificate, you are going to spend quite a bit of time revising. Spending a little time learning how to revise effectively will help you get more from your time and help you absorb and understand more of the material on the course. Here are some tips to help you revise for maths.

- Find a quiet place where you can work. This place should be dedicated to study, free of potential distractions. Turn off music, the TV, computer and mobile phone.
- Draw up a study plan. Don't be afraid to ask your parents/teachers/guidance counsellor for help at this stage.
- Do the more challenging revision first, when you are fresh. Trying to focus on difficult problems when you are tired can be counter-productive.
- Maths is based on understanding, so while you can learn some elements of the course, it is important that you develop an understanding of the material.
- Drill and practice are essential ingredients for success in maths.
- Try to link any new material to things you know already. This is learning through association and helps long-term retention.

A green circular icon with a yellow key inside, containing the text 'key point'.

Study in small chunks lasting 25 to 35 minutes. Your memory and concentration will work better if you study in short, frequent, bursts.

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Don't get hung up on more difficult material. Concentrate on understanding the fundamental concepts and being able to answer all of the straightforward questions. Then, with time, you can build up to the more challenging problems.

Leaving Certificate examination

Exam focus is critical to exam success. It is important to prepare yourself for the challenge you will face. By learning about the structure of the exam, you will learn how to maximise your points, allocate your time effectively and to manage the paper without panic.

The order of the questions is not set and some questions may include cross-syllabus topics. The examination paper will be presented in two sections, as follows:

Section A – 150 marks

Concepts and Skills

Section B – 150 marks

Contexts and Applications



Read the exam paper right through at the start in order to determine which question is the easiest one to begin with. Your mind may also be subconsciously processing some of the problems.



Start with your best question, then your next best and so on. This way, if you are short of time, at least your best questions will be done.

Time yourself as follows

- Reading the paper at the start: 5 minutes
- Section A: 70 minutes
- Section B: 70 minutes
- Reviewing your answers at the end: 5 minutes
- Try to stick closely to these times. If you run out of time on a question, leave it and come back at the end.



Rule of thumb for timing yourself during the exam:

Time spent on question = $\frac{1}{2}$ (marks for question)

That is, a 25-mark question should take no more than 12.5 minutes.

Further exam tips

- There is no such thing as ‘rough work’ in maths – all work is relevant. If the examiner doesn’t know how you reached an answer, even a correct answer, then full marks will not usually be awarded. Thus, **show all your work**.



- Attempt marks (partial credit) will be awarded for any step in the right direction. Therefore, **make an attempt at each part of the question**. Even if you do not get the correct answer, you can still pick up most of the marks on offer if you show how you worked it out. Also, **draw a diagram where possible**, because this can help you see the solution.
- If you cannot finish part of a question, leave a space and come back to it later. **Never scribble out any work or use Tipp-Ex**. Put a single line through it so that the examiner can still read it. In many cases, work that has a line through it can receive more marks than the candidate's other attempts.
- **Avoid using pencil** because the writing can be very faint and difficult to read.
- It is a good idea to show each stage of a calculation when using a calculator (in case you press a wrong key). Familiarise yourself with your calculator. Know your *booklet of formulae and tables* well and write down any formula that you use.



Attempt marks (partial credit) are valuable, so it is vital that you attempt all questions. Leave **NO** blanks.



Your calculator and *booklet of formulae and tables* are two extremely valuable resources to have in the exam. Make sure that you are very familiar with how your calculator works and that you know how to perform all functions on it. Also familiarise yourself with the *booklet of formulae and tables* so that you don't waste any time in the exam trying to find formulae.

Glossary of words used on the examination paper

Write down, state

You can write down your answer without showing any work. However, you can show some workings if you want to.

Calculate, find, show that, determine, prove

Obtain your answers by showing all relevant work. Marks are available for showing the steps leading to your final answer or conclusion.

Solve

Find the solution, or root, of an equation. The solution is the value of the variable that makes the left-hand side balance with the right-hand side.

Evaluate

Usually to work out, or find, a numerical value by putting in numbers for letters.

Comment on

After studying the given information or your answers, give your opinion on their significance.

Plot

Indicate the position of points on a graph, usually on the x - and y -planes.

Construct

Draw an accurate diagram, usually labelled, using a pencil, ruler, set square, compass and protractor. Leave all construction marks on your diagram.

Sketch

Make a rough diagram or graph, labelled if needed.

Hence

You *must* use the answer, or result, from the previous part of the question.

Hence or otherwise

It is recommended that you use the answer, or result, from the previous part of the question, but other methods are acceptable.

Syllabus and checklist for Leaving Certificate Ordinary Level Maths Paper 1 exam

The philosophy of your mathematics course is that topics can overlap, so you may encounter Paper 1 material on Paper 2 and vice versa.



Throughout your course you will be asked to apply your knowledge and skills to solve problems in familiar and unfamiliar contexts. In problem solving, you should use some of the following strategies:

- trial and improvement
- draw a diagram
- look for a pattern
- act it out
- draw a table
- simplify the problem
- use an equation
- work backwards
- eliminate possibilities.

The syllabus stresses that in all aspects of the Leaving Certificate Maths course, students should be able to:

- Explore patterns and formulate conjectures.
- Explain findings.
- Justify conclusions.
- Communicate mathematics verbally and in written form.

1

Algebra

aims

- To learn how to evaluate expressions
- To learn how to simplify algebraic expressions
- To learn how to factorise expressions
- To learn how to change the subject of a formula
- To learn how to work with irrational numbers

Evaluating expressions

To evaluate expressions, given the value of the variables, substitute the values in for the variables and evaluate the expression.

Example

Find the value of $5(2a - b)$ when $a = -3$ and $b = 7$.

Solution

$$\begin{aligned}
 &5(2a - b) \\
 &= 5(2(-3) - (7)) \quad (\text{let } a = -3 \text{ and } b = 7) \\
 &= 5(-6 - 7) \\
 &= 5(-13) \\
 &= -65
 \end{aligned}$$

key point

Take care when substituting in negative values. Always put brackets around these, then simplify the expression.

Example

Find the value of $p^2 - 6pq$ when $p = -2$ and $q = 3$.

Solution

$$\begin{aligned}
 &p^2 - 6pq \\
 &= (-2)^2 - 6(-2)(3) \quad (\text{let } p = -2 \text{ and } q = 3) \\
 &= 4 - 6(-6) \quad ((-2)^2 = (-2)(-2) = +4) \\
 &= 4 + 36 \\
 &= 40
 \end{aligned}$$

exam focus

Evaluating expressions is a vital skill for you to have throughout all aspects of your maths course.

exam
Q

Find the value of $\frac{2x - y + 3}{x^2 + 2y}$ when $x = \frac{3}{2}$ and $y = \frac{2}{3}$.

**Solution**

$$\frac{2x - y + 3}{x^2 + 2y}$$

$$= \frac{2\left(\frac{3}{2}\right) - \left(\frac{2}{3}\right) + 3}{\left(\frac{3}{2}\right)^2 + 2\left(\frac{2}{3}\right)} \quad \left(\text{let } x = \frac{3}{2} \text{ and } y = \frac{2}{3}\right)$$

$$= \frac{3 - \frac{2}{3} + 3}{\frac{9}{4} + \frac{4}{3}} \quad (\text{use calculator})$$

$$= \frac{\frac{16}{3}}{\frac{43}{12}} \quad (\text{add the fractions on the top and bottom})$$

$$= \frac{16}{3} \times \frac{12}{43} \quad (\text{turn the bottom fraction upside-down and multiply})$$

$$= \frac{192}{129}$$

$$= \frac{64}{43} \quad (\text{simplify by dividing 3 into the top and bottom})$$

Simplifying algebraic expressions

You must be able to apply the associative and distributive properties when simplifying algebraic expressions.

Associative property

$$(A \times B) \times C = A \times (B \times C)$$

Distributive property

$$A(B + C) = AB + AC$$

$$(A + B)(C + D) = A(C + D) + B(C + D)$$

exam
Q

(2015 Q.3 (a))

Simplify $3(4 - 5x) - 2(5 - 6x)$ **Solution**

$$3(4 - 5x) - 2(5 - 6x)$$

$$12 - 15x - 10 + 12x$$

$$2 - 3x$$

(multiply out the brackets)

(add like terms)

key
point

Add and subtract like terms.

ExampleSimplify $2a(4a + 3) - 4(3a - 7)$ **Solution**

$$2a(4a + 3) - 4(3a - 7)$$

$$8a^2 + 6a - 12a + 28$$

$$8a^2 - 6a + 28$$

(multiply out the brackets)

(add like terms)

ExampleSimplify $(2x + 3)(x - 4)$.**Solution****Method 1: use distributive law**

$$(2x + 3)(x - 4)$$

$$2x(x - 4) + 3(x - 4)$$

$$2x^2 - 8x + 3x - 12$$

$$2x^2 - 5x - 12$$

(remember $+3 \times -4 = -12$)

(multiply out the brackets)

(add like terms)

key
point

There are two methods for multiplying out brackets. These are both shown in this example. You can use whichever method you prefer.

Method 2: use box method

Put the terms in the first bracket on the top and terms from the second bracket down the side. Multiply each term by each other term.

	$2x$	$+3$
x	$2x^2$	$+3x$
-4	$-8x$	-12

Listing all terms from inside the boxes:

$$2x^2 - 8x + 3x - 12$$

$$2x^2 - 5x - 12$$

(add like terms)

ExampleSimplify $(3p + 2q)(p - 3q)$.**Solution**

$$(3p + 2q)(p - 3q)$$

$$3p(p - 3q) + 2q(p - 3q) \quad (\text{using distributive law})$$

$$3p^2 - 9pq + 2pq - 6q^2 \quad (\text{multiply out the brackets})$$

$$3p^2 - 7pq - 6q^2 \quad (\text{add like terms})$$

ExampleSimplify $(5a - 7)^2$.**Solution**

$$(5a - 7)^2 = (5a - 7)(5a - 7)$$

	$5a$	-7		
$5a$	$25a^2$	$-35a$	$25a^2 - 35a - 35a + 49$	(using box method)
-7	$-35a$	$+49$	$25a^2 - 70a + 49$	(add like terms)

Simplify $(3x - 4)(x^2 + 3x - 6)$.**Solution**

$$(3x - 4)(x^2 + 3x - 6)$$

$$3x(x^2 + 3x - 6) - 4(x^2 + 3x - 6)$$

$$3x^3 + 9x^2 - 18x - 4x^2 - 12x + 24 \quad (\text{multiply out the brackets})$$

$$3x^3 + 5x^2 - 30x + 24 \quad (\text{add like terms})$$

Adding algebraic fractions

You must put each fraction over the lowest common multiple (LCM) of the denominators. The LCM is the smallest number or expression that all denominators divide into evenly.

Example

Simplify $\frac{x+3}{2} + \frac{2x-1}{5}$.

Solution

$$\frac{5(x+3) + 2(2x-1)}{10} \quad (\text{LCM of the denominators 2 and 5 is 10})$$

$$\frac{5x+15+4x-2}{10} \quad (\text{multiply out brackets})$$

$$\frac{9x+13}{10} \quad (\text{simplify})$$



Simplify $\frac{x-4}{3} - \frac{3x-1}{6} + \frac{2x+5}{4}$.

**Solution**

$$\frac{4(x-4) - 2(3x-1) + 3(2x+5)}{12} \quad (\text{LCM of the denominators 3, 6 and 4 is 12})$$

$$\frac{4x-16-6x+2+6x+15}{12} \quad (\text{multiply out brackets})$$

$$\frac{4x+1}{12} \quad (\text{simplify})$$

Example

Simplify $\frac{3}{p+2} + \frac{5}{p-1}$.

Solution

$$\frac{3(p-1) + 5(p+2)}{(p+2)(p-1)} \quad (\text{LCM of the denominators } (p+2) \text{ and } (p-1) \text{ is } (p+2)(p-1))$$

$$\frac{3p-3+5p+10}{(p+2)(p-1)} \quad (\text{multiply out brackets})$$

$$\frac{8p+7}{(p+2)(p-1)} \quad (\text{simplify})$$

Example

Simplify $\frac{2}{x-3} - \frac{7}{2x+5} + \frac{2}{3}$.

Solution

Common denominator of $(x-3)$, $(2x+5)$ and 3 is $(x-3)(2x+5)(3)$:

$$\frac{2(2x+5)(3) - 7(x-3)(3) + 2(x-3)(2x+5)}{(x-3)(2x+5)(3)}$$

$$\frac{2(6x+15) - 7(3x-9) + 2(2x^2+5x-6x-15)}{(x-3)(2x+5)(3)} \quad (\text{multiply out brackets on top})$$

$$\frac{12x+30-21x+63+2(2x^2-1x-15)}{(x-3)(2x+5)(3)} \quad (\text{multiply out brackets on top})$$

$$\frac{-9x+93+4x^2-2x-30}{(x-3)(2x+5)(3)} \quad (\text{multiply out brackets on top})$$

$$\frac{4x^2-11x+63}{(x-3)(2x+5)(3)} \quad (\text{simplify})$$

Factorising and simplifying expressions

There are four types of factorising that we will meet on this course:

<p>Take out common terms</p> $ab + ad = a(b + d)$	<p>Factorising by grouping</p> $ab + ad + cb + cd = (a + c)(b + d)$
<p>Factorise a trinomial</p> $a^2 - 2ab + b^2 = (a - b)(a - b)$	<p>Difference of two squares</p> $a^2 - b^2 = (a + b)(a - b)$



Factorising is a basic and vital skill for you to have throughout your maths course. You must be able to factorise expressions quickly and easily. This will take practice, but it is worth spending time on.

1. Take out common terms

1. Find the highest common factor (HCF) of all the terms making up the expression. This is the biggest value (constants or variables) that divides into all terms evenly.
2. Put the HCF outside the brackets.
3. Divide each term by the HCF to find the factor inside the brackets.

Example

Factorise the following:

$$(i) 3p^2 + 6pq$$

$$(ii) ab - 2a^2b + 3ab^2$$

Solution

$$(i) 3p^2 + 6pq \quad (\text{HCF is } 3p)$$

$$3p(p + 2q) \quad (\text{factorise out } 3p)$$

$$(ii) ab - 2a^2b + 3ab^2$$

$$ab - 2aab + 3abb \quad (\text{HCF is } ab)$$

$$ab(1 - 2a + 3b) \quad (\text{factorise out } ab)$$



2. Factorising by grouping

Use this method when you have four terms, with no common factor.

1. Group into pairs with a common factor.
2. Take out the HCF in each pair separately.
3. Take out the new common factor.

Example

Factorise the following:

$$(i) 3pr - 3ps + qr - qs$$

$$(ii) a^2 + xy - ay - ax$$

Solution

$$(i) 3pr - 3ps + qr - qs \quad (\text{already in pairs with a common factor})$$

$$3p(r - s) + q(r - s) \quad (\text{take out common factor in each pair})$$

$$(r - s)(3p + q) \quad (\text{take out the common factor})$$

