Business Calculations
Euro Edition

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Basic arithmetical operations

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FRACTIONS

A fraction is a part of a whole number. For example, $\frac{5}{8}$ of an orange means that an orange has been divided into 8 parts and that we are interested in 5 of these parts. Fractions are part of everyday life, for example, we talk about $\frac{1}{2}$ of something, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{8}$ etc.

Fractions can be added, subtracted, multiplied and divided.

All fractions have two parts, separated by a line. The top part of the fraction is called the *numerator*. The lower part of the fraction is called the *denominator*. In the case of the fraction $\frac{5}{8}$, 5 is the numerator and 8 is the denominator.

A fraction where the numerator is smaller than the denominator is called a *proper fraction*. Examples are $\frac{4}{8}$ and $\frac{3}{8}$.

A fraction where the numerator is greater than the denominator is called an *improper fraction*. (It is sometimes also called a top heavy fraction). Examples are $\frac{9}{8}$ and $\frac{14}{8}$.

A number made up of whole numbers and fractions is called a *mixed number*. Examples are $1\frac{4}{5}$ and $3\frac{6}{7}$.

Note: A fraction should always be given in its lowest form. This means that before we add or subtract fractions, we should find the lowest common denominator or the lowest common multiple (LCM). This is the lowest whole number (integer) into which the denominator of the fractions will go evenly.

Example 1

Find the lowest common multiple of $\frac{1}{3} + \frac{2}{9} + \frac{3}{18}$.

Answer

Use the following procedure to get the LCM.

1. Look at the biggest denominator – in this case 18.
2. Will the other denominators divide into it exactly? In this case will 3 and 9 divide into 18 exactly? Yes, they will.
3. Therefore the LCM is 18.

Addition of fractions

Example 2

Add $\frac{1}{3} + \frac{2}{9} + \frac{3}{18}$.

Answer

First we must find the LCM, which is 18 (see previous section).

The next step is to divide each denominator into the LCM and multiply the corresponding numerator by the result. Therefore $\frac{1}{3}$ becomes $\frac{6}{18}$, $\frac{2}{9}$ becomes $\frac{4}{18}$ and $\frac{3}{18}$ remains $\frac{3}{18}$.

We can now add the fractions in the new form as follows:

$$\frac{6}{18} + \frac{4}{18} + \frac{3}{18}$$

$$= \frac{6+4+3}{18}$$

$= \frac{13}{18}$ (this is a *proper fraction*)

Example 3

Add $\frac{2}{5} + \frac{6}{8} + \frac{1}{20}$.

Answer

Follow the procedure above to obtain the LCM. The biggest numerator is 20. The other numerators 5 and 8 will not divide into 20 exactly. The next step is to find a multiple of 20 that 5 and 8 will divide into exactly. We do this by going upwards through the 20 times table to find a number that 5 and 8 will divide into exactly.

$$20 \times 2 = 40$$

5 and 8 will divide into 40
Therefore our LCM is 40.
Now divide each denominator in turn into 40 and multiply the corresponding numerator by the result

$$= \frac{16 + 30 + 2}{40}$$

$= \frac{48}{40}$ (this is an *improper fraction*)
$= 1\frac{8}{40}$ (this is a *mixed number*)

Addition of mixed numbers

Example 4

Add $1\frac{2}{3} + 6\frac{1}{3} + 1\frac{2}{9}$.
Note: We are now dealing with whole numbers as well as fractions.

Answer

First add the whole numbers = 1 + 6 + 1 = *8
Now add $\frac{2}{3} + \frac{1}{3} + \frac{2}{9}$ using the method in Example 2
LCM is 9

$$= \frac{6 + 3 + 2}{9}$$

$= \frac{11}{9}$

This gives $1\frac{2}{9}$
Total is $*8 + 1\frac{2}{9} = 9\frac{2}{9}$
We could have solved this exercise in an alternative way. We could have turned each of the mixed numbers into an improper fraction and added these using the method in Example 2.

Alternative answer

$1\frac{2}{3} + 6\frac{1}{3} + 1\frac{2}{9}$
Convert the mixed numbers into improper fractions.
To do this first multiply the whole number (integer) by the denominator, add the result to the numerator and place the answer over the denominator.
Therefore $1\frac{2}{3}$ becomes $\frac{5}{3}$
$6\frac{1}{3}$ becomes $\frac{19}{3}$
$1\frac{2}{9}$ becomes $\frac{11}{9}$. Now the question is $\frac{5}{3} + \frac{19}{3} + \frac{11}{9}$

LCM is 9

$$= \frac{15 + 57 + 11}{9}$$

$$= \frac{83}{9}$$

9 will go into 83 9 times with $\frac{2}{9}$ over

$$= 9\frac{2}{9}.$$

Exercise 1.1

1. Add the following fractions:
   (a) $\frac{1}{2} + \frac{2}{3} =$
   (b) $\frac{2}{3} + \frac{1}{4} + \frac{1}{2} =$
   (c) $\frac{2}{3} + \frac{1}{5} =$
   (d) $\frac{3}{20} + \frac{4}{5} + \frac{5}{8} =$
   (e) $\frac{1}{2} + \frac{3}{4} + \frac{3}{5} + \frac{5}{8} =$
   (f) $\frac{6}{7} + \frac{2}{3} + \frac{9}{21} =$

2. Add the following mixed fractions:
   (a) $14\frac{1}{2} + 3\frac{4}{6} + \frac{5}{9} =$
   (b) $2\frac{5}{8} + 2\frac{6}{3} + 3\frac{9}{12} =$
   (c) $3\frac{2}{7} + 5\frac{1}{3} + 3\frac{15}{21} =$
   (d) $5\frac{4}{12} + 2\frac{4}{6} + 4\frac{1}{4} =$
   (e) $10\frac{1}{2} + 5\frac{6}{10} + 3\frac{1}{5} =$
   (f) $5\frac{1}{6} + 4\frac{9}{12} + 4\frac{19}{36} =$

Subtraction of fractions

Fractions are subtracted using the same procedure as for addition.
The first step is to find the LCM and then convert each of the fractions to this denominator. Then subtract the numerators. If necessary, cancel the resulting fraction to its lowest form.

Example 5

Subtract $\frac{3}{4} - \frac{1}{8}$

Answer

LCM is 8.

$$\frac{6 - 1}{8} = \frac{5}{8}$$

Subtracting mixed numbers

Example 6

Subtract $8\frac{1}{2} - 1\frac{1}{5}$

Answer

$$
\begin{aligned}
& 8\tfrac{1}{2} - 1\tfrac{1}{5} \\
&= 7\tfrac{1}{2} - \tfrac{1}{5} \\
&= \tfrac{15}{2} - \tfrac{1}{5} \\
&= \frac{75 - 2}{10} \\
&= \tfrac{73}{10} \\
&= 7\tfrac{3}{10}
\end{aligned}
$$

Exercise 1.2

1. Subtract the following fractions:
   (a) $\frac{2}{3} - \frac{1}{9} =$
   (b) $\frac{5}{6} - \frac{3}{8} =$
   (c) $\frac{5}{8} - \frac{1}{3} =$
   (d) $\frac{13}{15} - \frac{3}{10} =$
   (e) $\frac{1}{4} - \frac{1}{8} =$
   (f) $\frac{4}{5} - \frac{8}{15} =$

2. Subtract the following fractions:
   (a) $1\frac{7}{10} - 1\frac{1}{2} =$
   (b) $2\frac{9}{11} - 1\frac{4}{11} =$
   (c) $14\frac{3}{10} - 8\frac{2}{5} =$
   (d) $8\frac{7}{8} - 5\frac{4}{5} =$
   (e) $6\frac{3}{4} - 3\frac{2}{5} =$
   (f) $15\frac{3}{7} - 7\frac{4}{5} =$

Multiplication of fractions

The LCM is not needed to multiply fractions. Simply multiply the numerators by each other and similarly the denominators by each other. Look at the following example.

Example 7

Multiply $\frac{4}{8} \times \frac{12}{16} \times \frac{3}{4}$

Answer

$$\frac{4 \times 12 \times 3}{8 \times 16 \times 4}$$

$$= \frac{144}{512}$$

We can simplify this fraction by dividing the numerator and denominator by 8, i.e. we have found a number which divides into both numerator and denominator evenly

$$= \frac{18}{64}$$

This can be further simplified by dividing the top and bottom by 2.
Therefore the answer $= \frac{9}{32}$.
This process is called *cancelling*. We could have started solving the above example in the first place by cancelling the figure 4 below the line with the figure 4 above the line. Please see the next example.

Example 8

Multiply $\frac{4}{8} \times \frac{12}{16} \times \frac{3}{4}$.

Answer

Cancel the 4 above the line with the 4 below the line twice

$$= \frac{\cancel{4}^{1}}{8} \times \frac{\cancel{12}^{3}}{\cancel{16}_{4}} \times \frac{3}{\cancel{4}_{1}} = \frac{9}{32}$$

Note: The value of a fraction is not changed if the numerator and denominator are each multiplied or divided by the same number.

Exercise 1.3

1. Multiply these fractions:
   (a) $\frac{1}{3} \times \frac{3}{4} =$
   (b) $\frac{3}{7} \times \frac{5}{8} =$
   (c) $\frac{1}{3} \times \frac{7}{10} =$
   (d) $\frac{3}{8} \times \frac{5}{9} =$
   (e) $\frac{17}{20} \times \frac{5}{16} =$
   (f) $\frac{14}{15} \times \frac{9}{12} =$

2. Multiply these fractions
   (a) $\frac{7}{11} \times \frac{9}{10} \times \frac{20}{22} =$
   (b) $\frac{6}{7} \times \frac{8}{9} \times \frac{16}{17} =$
   (c) $\frac{2}{7} \times \frac{3}{5} \times \frac{15}{16} =$
   (d) $\frac{3}{4} \times \frac{19}{21} \times \frac{6}{10} =$
   (e) $\frac{7}{10} \times \frac{4}{15} \times \frac{15}{16} =$
   (f) $\frac{11}{12} \times \frac{3}{4} \times \frac{4}{9} =$

Multiplication of mixed numbers

The best way to multiply mixed numbers is by first changing them into improper fractions. Then multiply the numerators and denominators in the usual way, cancelling down if possible. If the result is itself an improper fraction, then convert it into a mixed number.

Example 9

Multiply $2\frac{5}{8} \times 3\frac{2}{3}$.

Answer

First change into improper fractions

$$= \frac{21}{8} \times \frac{11}{3}$$

Now cancel above and below by 3

$$= \frac{\cancel{21}^{7}}{8} \times \frac{11}{\cancel{3}_{1}}$$

$$= \frac{7}{8} \times \frac{11}{1}$$

$$= \frac{77}{8}$$

Now convert this to a mixed number

$$= 9\frac{5}{8}$$

Exercise 1.4

1. Multiply these mixed numbers
   (a) $2\frac{1}{2} \times 3\frac{1}{3} =$
   (b) $10\frac{1}{4} \times 11\frac{1}{7} =$
   (c) $2\frac{3}{4} \times 3\frac{2}{11} =$
   (d) $2\frac{1}{3} \times 2\frac{3}{4} \times 2\frac{2}{9} =$
   (e) $5\frac{2}{3} \times 1\frac{1}{2} \times 1\frac{2}{31} =$
   (f) $3\frac{7}{8} \times 3\frac{1}{3} \times 1\frac{1}{5} =$
   (g) $1\frac{7}{8} \times 3\frac{3}{5} \times 2\frac{1}{7} =$

Division of fractions

The rule for dividing fractions is to *turn the divisor upside down and multiply*.

Example 10

Divide 20 by $\frac{1}{4}$.

Answer

$$20 \div \tfrac{1}{4}$$
$$= \tfrac{20}{1} \times \tfrac{4}{1}$$
$$= 80$$

Example 11

Divide $\frac{1}{3}$ by $\frac{5}{8}$.

Answer

$$\tfrac{1}{3} \div \tfrac{5}{8}$$
$$= \tfrac{1}{3} \times \tfrac{8}{5}$$
$$= \tfrac{8}{15}$$

Example 12

Divide $\frac{3}{4}$ by 2.

Answer

$$\tfrac{3}{4} \div 2$$
$$= \tfrac{3}{4} \div \tfrac{2}{1}$$
$$= \tfrac{3}{4} \times \tfrac{1}{2}$$
$$= \tfrac{3}{8}$$

Dividing mixed fractions

To divide mixed fractions simply change them into improper fractions, turn the divisor upside down and multiply as before.

Example 13

Divide $2\frac{3}{4}$ by $1\frac{1}{3}$.

Answer

$2\frac{3}{4} \div 1\frac{1}{3}$

$= \frac{11}{4} \div \frac{4}{3}$
$= \frac{11}{4} \times \frac{3}{4}$
$= \frac{33}{16}$
$= 2\frac{1}{16}$

Exercise 1.5

1. Divide the following fractions:
   (a) $\frac{3}{4} \div \frac{3}{8} =$
   (b) $\frac{2}{9} \div \frac{18}{19} =$
   (c) $\frac{4}{11} \div \frac{11}{16} =$
   (d) $\frac{3}{9} \div \frac{9}{27} =$
   (e) $\frac{11}{13} \div \frac{4}{7} =$
   (f) $\frac{22}{31} \div \frac{19}{20} =$

2. Divide the following mixed fractions:
   (a) $1\frac{2}{3} \div 2\frac{3}{8} =$
   (b) $16\frac{1}{7} \div 1\frac{2}{7} =$
   (c) $19\frac{1}{7} \div 21\frac{1}{6} =$
   (d) $2\frac{5}{8} \div 3\frac{6}{7} =$
   (e) $9\frac{7}{9} \div 6\frac{7}{21} =$
   (f) $23\frac{1}{3} \div 5\frac{1}{4} =$

Use of the word ‘of’

Note: Sometimes the word ‘of’ can cause confusion in mathematics. In mathematics ‘of’ means ‘multiplied by’. See the following examples.

Example 14

Find $\frac{1}{2}$ of 8.

Answer

$\frac{1}{2} \times \frac{8}{1} = 4$

Example 15

Find $\frac{1}{4}$ of 12.

Answer

$\frac{1}{4} \times \frac{12}{1} = 3$

Example 16

Find $\frac{5}{6}$ of $3\frac{3}{10}$.

Answer

$\frac{5}{6} \times \frac{33}{10}$

Cancel 3 into 33 and 6, cancel 5 into 5 and 10

$= \frac{1}{2} \times \frac{11}{2}$
$= \frac{11}{4}$
$= 2\frac{3}{4}$

Exercise 1.6

1. (a) Find $\frac{1}{3}$ of 9.
   (b) Find $\frac{1}{2}$ of 240.
   (c) Find $\frac{1}{4}$ of 200.
   (d) Find $\frac{1}{8}$ of 16.

Calculations involving combinations of arithmetic operations

Sometimes a question may include addition, subtraction, multiplication and division.
Note: It is very important to remember that multiplication and division must be done before addition and subtraction.

Example 17

Solve the following: $2\frac{2}{5}\,(4\frac{2}{3} + 2\frac{5}{6}) - 3\frac{3}{4} \div 2\frac{1}{2}$

Answer

$2\frac{2}{5}\,(6\frac{2}{3} + \frac{5}{6}) - \frac{15}{4} \div \frac{5}{2}$

$= \frac{12}{5}\left(6\,\frac{4+5}{6}\right) - \frac{\cancel{15}^{3}}{\cancel{4}_{2}} \times \frac{\cancel{2}^{1}}{\cancel{5}_{1}}$

$= \frac{12}{5} \times 7\frac{1}{2} - \frac{3}{2}$

$= \frac{\cancel{12}^{6}}{\cancel{5}_{1}} \times \frac{\cancel{15}^{3}}{\cancel{2}_{1}} - \frac{3}{2}$

$= 18 - 1\frac{1}{2}$
$= 16\frac{1}{2}$

Exercise 1.7

1. (a) $\frac{3}{4}(\frac{1}{3}+\frac{2}{5})$
   (b) $1\frac{1}{3}(3\frac{1}{3}+6\frac{1}{9})-1\frac{1}{3}$
   (c) $(6\frac{1}{4}+4\frac{3}{8})\times(\frac{2}{9}-\frac{1}{18})$
   (d) $(9\frac{2}{5}\div\frac{1}{10})-1\frac{1}{5}(6\frac{2}{3}+4\frac{5}{6})$
   (e) $21\frac{1}{4}-\frac{1}{8}(\frac{1}{16}+\frac{2}{24})$

DECIMALS

We now know that a fraction is a part of a unit and that this is expressed by placing one number (the numerator) over another (the denominator), e.g. $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$.
There is another way in which a fraction may be expressed, i.e. by using the decimal system.
The word decimal comes from the Latin word *decem* which means 10.
A decimal fraction is so-called because it is expressed as part of 10, or as a fraction of a multiple of 10. For example, $\frac{5}{100}$, $\frac{25}{100}$, etc.
The *decimal point* is a dot which separates the whole number from the part number. After the decimal point, the first place is for $\frac{1}{10}$ths, the second place is for $\frac{1}{100}$ths etc.
For example, if we take 2.186, the digit 1 has value $\frac{1}{10}$, the digit 8 has the value $\frac{8}{100}$ and the digit 6 has the value $\frac{6}{1000}$.

Converting decimals to fractions

Decimals may be converted into fractions as shown in the following example.

Example 18

Convert 1.186 to a fraction.

Answer

$$1.186 = 1+\frac{1}{10}+\frac{8}{100}+\frac{6}{1000} = 1\frac{186}{1000}$$

Example 19

Convert 6.8 to a fraction.

Answer

$$6.8 = 6+\frac{8}{10}$$

Now divide 8 and 10 by 2

$$= 6+\frac{4}{5} = 6\frac{4}{5}$$

Example 20

Convert 12.11 to a fraction.

Answer

$12.11 = 12 + \frac{1}{10} + \frac{1}{100} = 12 + \frac{11}{100} = 12\frac{11}{100}$.

Converting fractions to decimals

Fractions can be converted to decimals by dividing the denominator into the numerator, as shown in the following examples.

Example 21

Convert $\frac{3}{5}$ to a decimal.

Answer

Divide 5 into 3 $\quad 5\overline{)3.0} = 0.60.$

Example 22

Convert $\frac{4}{100}$ to a decimal.

Answer

Divide 100 into 4 $\quad 100\overline{)4.00} = 0.04.$

Example 23

Convert $\frac{6}{1000}$ to a decimal.

Answer

Divide 1000 into 6 = 0.006.

Note: Be very careful with the placing of the decimal point.

Exercise 1.8

1. What is the value of 6 in each of these? (Express each answer as a fraction.)
   (a) 1.675
   (b) 1.764
   (c) 8.756
   (d) 88.62

2. Write these as fractions in their lowest form:
   (a) 0.6
   (b) 0.7
   (c) 0.06
   (d) 0.007
   (e) 1.5
   (f) 0.16

3. Write these as decimals:
   (a) $\frac{6}{10}$
   (b) $\frac{7}{100}$
   (c) $\frac{4}{1000}$
   (d) $\frac{12}{100}$
   (e) $\frac{21}{1000}$
   (f) $4\frac{1}{10}$

Addition and subtraction of decimals

When adding and subtracting numbers which have decimal fractions, *always keep the decimal points underneath one another*.
This brings units under units, tens under tens, hundreds under hundreds on one side of the decimal point; on the other side, the tenths will be under the tenths, the hundredths under hundredths etc.

Example 24

Add the following numbers: 1.16, 1.176, 2.89 and 6.742.

Answer

Note: It is advisable to place the largest number on top.

$$\begin{array}{r} 6.742 \\ 2.89 \\ 1.176 \\ 1.116 \\ \hline 11.968 \end{array}$$

Example 25

Subtract 6.84 from 64.241.

Answer

$$\begin{array}{r} 64.241 \\ -6.84 \\ \hline 57.401 \end{array}$$

Exercise 1.9

1. Add the following:
   (a) 62.48 + 16.50 + 16.99 + 27.94 =
   (b) 45.42 + 64.99 + 31.71 + 68.49 =
   (c) 124.68 + 1.72 + 19.624 + 9.024 =
   (d) 897.02 + 999.99 + 178.95 + 263.974 =
   (e) 1987.5 + 2977.48 + 2.972 + 6.5 =
   (f) 589.62 + 997.456 + 2987.70 + 21.5 =

2. Subtract the following:
   (a) 9.7 – 6.25 =
   (b) 68.72 – 45.5 =
   (c) 148.64 – 27.829 =
   (d) 998.25 – 2.786 =
   (e) 32 900 – 16.7821 =
   (f) 9764.21 – 84.666 =

Multiplication of decimals

The easiest way of multiplying decimals is to ignore the decimal points altogether until the end of the multiplication and only then to find the correct position of the decimal point in the answer.

Example 26

Multiply 2.68 by 2.1.

Answer

Step 1: Ignore the decimal points and carry out the multiplication in the normal way.

```
  2.68 ×
   2.1
 ------
   268
 5360
 ------
 5.628
```

Step 2: To find the position of the decimal point carry out the following procedure. Count the places after the decimal point in *both* sets of figures; there will be the same number of places after the decimal point in the answer.
In the above example there is one place after the decimal point in 2.1 and two places after the decimal point in 2.68. This means that there are three places after the decimal point altogether. Therefore there will be three places after the decimal point in the answer. The answer therefore is 5.628.

Exercise 1.10

1. Multiply the following:
   (a) 6.82 × 2.1 =
   (b) 8.97 × 6.5 =
   (c) 97.2 × 28.1 =
   (d) 269.786 × 0.928 =
   (e) 9.66 × 2.9 =
   (f) 672.5 × 99.04 =
   (g) 729.4 × 0.52 =
   (h) 628.997 × 0.624 =
   (i) 829.7 × 1.97 =

Multiplying and dividing decimals by multiples of 10

To multiply by 10, move the decimal point 1 place to the right;
to multiply by 100 move the decimal point 2 places to the right;
to multiply by 1000 move the decimal point 3 places to the right and so on for further multiples of 10.
The opposite applies when dividing by 10, 100 etc., i.e. the decimal point is moved to the left.

Example 27

Multiplication

$2.689 \times 10 = 26.89$
$2.689 \times 100 = 268.9$
$2.689 \times 1000 = 2\ 689.$

Division

$2.689 \div 10 = 0.2689$
$2.689 \div 100 = 0.02689$
$2.689 \div 1\ 000 = 0.002689$

Exercise 1.11

1. Multiply the following:
   (a) $62.7 \times 10 =$
   (b) $42.976 \times 10 =$
   (c) $629.1 \times 10 =$
   (d) $1285.268 \times 10 =$
   (e) $129.42 \times 100 =$
   (f) $26.748 \times 1000 =$

2. Divide the following:
   (a) $62.97 \div 10 =$
   (b) $697.24 \div 100 =$
   (c) $697.24 \div 1000 =$
   (d) $298.2 \div 10 =$
   (e) $67.24 \div 1000 =$
   (f) $1976.1 \div 100 =$

Division of decimals

The simplest way to divide by a decimal is to change the divisor so that it becomes a whole number, e.g. $5.2 \div 2.6$. The only way to remove the decimal point in this case is to multiply by 10. Thus $10 \times 2.6 = 26$. But in order to compensate for this change we must do the same to the dividend, i.e. $5.2 \times 10 = 52$. Therefore the calculation now becomes $52 \div 26 = 2$.
Note: To divide decimals, change the divisor to a whole number, and move the decimal point the same number of places in the dividend.

Example 28

Find 128.674 ÷ 6.7.

Answer

First change the divisor to a whole number

i.e. $6.7 \times 10 = 67$

Now we must also multiply the dividend by 10, i.e. $128.674 \times 10 = 1286.74$
Therefore the question is now $1286.74 \div 67$

$= 19.20574$

Note 1: To 2 decimal places the answer is 19.21.
Note 2: Some divisions do not work out exactly, but the division process goes on and on.

Example 29

Find 100 ÷ 3.

Answer

$100 \div 3 = 33.333 \ldots$

The figure 3 recurs forever. This is known as a recurring decimal.

Example 30

Express $\frac{3}{7}$ as a decimal.

Answer

$3 \div 7 = 0.4285$

In divisions like these, the answer is usually required to a certain degree of accuracy, for example, correct to 2 decimal places. If the calculation comes to 68.4967, accuracy to $\frac{1}{100}$ of a unit gives an answer of 68.50.

Rounding off and dividing decimals

Exercise 1.12

1. Round off each of the following to 2 decimal places:
   (a) 64.703
   (b) 157.3562
   (c) 105.986
   (d) 1.085

2. Divide the following correct to 2 decimal places:
   (a) $2.5 \div 1.6 =$
   (b) $98.62 \div 7.9 =$
   (c) $45.789 \div 2.7 =$
   (d) $667.2 \div 9.6 =$
   (e) $1\ 297.1 \div 2.5 =$
   (f) $976.256 \div 2.9 =$
   (g) $72.77 \div 5.9 =$
   (h) $66.741 \div 2.96 =$
   (i) $1267.42 \div 29.74$

PERCENTAGES

The word *percentage* comes from the Latin word for 100, *centum*. Per means 'out of'. Therefore percentage means 'out of 100'. The % sign is used for 'per cent', so that, for example, 20 per cent = 20%.
If you are told that in a school, 20% of the students come from a rural background, then this means that out of every 100 pupils in the school, 20 of them have a rural background.

Frequently used percentages

$\frac{3}{4} = \frac{75}{100} = 75\%$
$\frac{1}{2} = \frac{50}{100} = 50\%$
$\frac{1}{4} = \frac{25}{100} = 25\%$

$$\frac{1}{3} = \frac{33\frac{1}{3}}{100} = 33\frac{1}{3}\%$$

$$\frac{2}{3} = \frac{66\frac{2}{3}}{100} = 66\frac{2}{3}\%$$

$\frac{1}{5} = \frac{20}{100} = 20\%$
$\frac{2}{5} = \frac{40}{100} = 40\%$
$\frac{1}{20} = \frac{5}{100} = 5\%$

$$\frac{1}{8} = \frac{12\frac{1}{2}}{100} = 12\frac{1}{2}\%$$

$$\frac{1}{40} = \frac{1}{40} \times 100 = \frac{10}{4} = 2\frac{1}{2}\%$$

Note: Percentages are used:

(a) to make comparisons between numbers
(b) to express relationships between numbers.

Conversion of fractions and decimals to percentages

Fractions, decimals and percentages are frequently used in business. Therefore you should become familiar with their conversions.
Note: To change a fraction to a percentage, put down the fraction and multiply it by 100, as shown in the following examples.

Example 31

Change $\frac{1}{2}$ to a percentage.

Answer

$$\frac{1}{\cancel{2}_{1}} \times \frac{\cancel{100}^{50}}{1} = 50\%$$

Example 32

Change $\frac{1}{4}$ to a percentage.

Answer

$$\frac{1}{\cancel{4}_{1}} \times \frac{\cancel{100}^{25}}{1} = 25\%$$

Example 33

Change $\frac{3}{4}$ to a percentage.

Answer

$$\frac{3}{\cancel{4}_{1}} \times \frac{\cancel{100}^{25}}{1} = 75\%$$

Note: To change a decimal to a percentage multiply the decimal by 100 and put the % sign in the answer, as shown in the following examples.

Example 34

Change 0.50 to a percentage.

Answer

$0.50 \times 100 = 50\%$

Example 35

Change 0.25 to a percentage.

Answer

$0.25 \times 100 = 25\%$

Example 36

Change 0.2987 to a percentage.

Answer

$0.2987 \times 100 = 29.87\%$

Example 37

Change 1.25 to a percentage.

Answer

$1.25 \times 100 = 125\%$

Exercise 1.13

1. Convert the following fractions to percentages:
   (a) $\frac{5}{8}$
   (b) $\frac{3}{4}$
   (c) $\frac{2}{5}$
   (d) $\frac{9}{8}$
   (e) $\frac{2}{20}$
   (f) $\frac{1}{8}$
   (g) $\frac{1}{16}$
   (h) $1\frac{6}{16}$
   (i) $5\frac{7}{8}$
   (j) $8\frac{1}{2}$
   (k) $4\frac{2}{5}$

2. Convert the following decimals to percentages:
   (a) 0.52
   (b) 1.25
   (c) 0.6789
   (d) 6.27
   (e) 2.9678
   (f) 0.256
   (g) 0.976
   (h) 2.87

Conversion of percentages to fractions and decimals

Note: To convert a percentage to a fraction put down the whole number of the percentage as the numerator, and put down 100 as the denominator. Then cancel if possible, as shown in the examples below.

Example 38

Convert 74% to a fraction.

Answer

$$74\% = \frac{\cancel{74}^{37}}{\cancel{100}_{50}} = \frac{37}{50}$$

Example 39

Convert 25% to a fraction.

Answer

$$25\% = \frac{\cancel{25}^{1}}{\cancel{100}_{4}} = \frac{1}{4}$$

Example 40

Convert 75% to a fraction.

Answer

$$75\% = \frac{\cancel{75}^{3}}{\cancel{100}_{4}} = \frac{3}{4}$$

Example 41

Convert 120% to a fraction.

Answer

$$120\% = \frac{12\cancel{0}}{10\cancel{0}} = \frac{\cancel{12}^{6}}{\cancel{10}_{5}} = \frac{6}{5} = 1\tfrac{1}{5}$$

Example 42

Convert 20.5% = to a fraction.

Answer

$$20.5\% = \frac{20\frac{1}{2}}{100} = \frac{41}{200}$$

Note: To change a percentage to a decimal you put down the percentage and divide it by 100, as shown in the following examples.

Example 43

Convert 25% to a decimal.

Answer

$25\% = 25.00 \div 100 = \frac{25}{100} = 0.25$

Example 44

Convert 85% to a decimal.

Answer

$85\% = 85.00 \div 100 = \frac{85}{100} = 0.85$

Example 45

Convert 21.25% to a decimal.

Answer

$21.25\% = 21.25 \div 100 = \frac{21.25}{100} = 0.2125$

Example 46

Convert 125% to a decimal.

Answer

$125\% = 125 \div 100 = \frac{125}{100} = 1.25$

Exercise 1.14

1. Convert the following percentages to fractions in their lowest form.
   (a) 20%
   (b) 50%
   (c) 56%
   (d) 48%
   (e) $27\frac{1}{4}\%$
   (f) 80%
   (g) $23\frac{1}{3}\%$
   (h) $12\frac{5}{11}\%$
   (i) 124%

2. Convert the following percentages to decimals.
   (a) 27%
   (b) 9.27%
   (c) 64%
   (d) 66.66%
   (e) 78%
   (f) 21%
   (g) 45%
   (h) 13%
   (i) 18.72%

Expressing one quantity as a percentage of another

Note: To express one quantity as a percentage of another, use the following:

$$\frac{\text{First quantity}}{\text{Second quantity}} \times \frac{100}{1}\,\%$$

Both quantities must be similar.

Example 47

What percentage is €10 of €20?

Answer

€10 is $\frac{10}{20}$ of €20

$\frac{10}{20}$ as a percentage is $\frac{10}{20} \times \frac{100}{1} = \frac{10}{1} \times \frac{5}{1} = 50\%$

Note: The smaller quantity is the numerator. The larger quantity is the denominator.

Exercise 1.15

1. Express the first quantity as a percentage of the second in each of the following.
   (a) 13, 50
   (b) 20 cents, €4
   (c) 28 cents, €1.40
   (d) 3, 4
   (e) €64, €100
   (f) 50 cents, €150
   (g) 10, 270
   (h) 70, 1 200
   (i) €45, €70

To find a percentage of a quantity

Note: To find a percentage of a quantity, write down the percentage required and multiply it by the original quantity.

Example 48

Find 6% of 2748.

Answer

6% of $2748 = \frac{6}{100} \times \frac{2748}{1} = 16\frac{488}{100} = 164.88$

Example 49

Find $12\frac{1}{2}\%$ of 567.

Answer

$12\frac{1}{2}\%$ expressed as a fraction is $\dfrac{12\frac{1}{2}}{100}$

Now change the whole number $= \dfrac{12\frac{1}{2} \times 2}{100 \times 2} = \dfrac{25}{200} = \frac{1}{8}$

$\frac{1}{8}$ of $567 = \frac{1}{8} \times \frac{567}{1} = 70.88$

Exercise 1.16

1. Find:
   (a) 20% of 968
   (b) 10% of 276
   (c) 50% of 1972
   (d) 90% of 29 763
   (e) 45% of €164
   (f) 25% of €742
   (g) 15% of €16 423
   (h) 21% of €70
   (i) 18% of 16
   (j) 75% of €92
   (k) 85% of 92
   (l) 92% of €60

2. Calculate the following percentage parts:
   (a) 52% of 480 kg
   (b) 25% of 50 km
   (c) 90% of 2000 litres
   (d) 50% of 870 litres
   (e) 40% of 800 km
   (f) 97% of 87 litres
   (g) 27% of 600 kg
   (h) 60% of 127 km

To find a number when given a percentage of that number

Example 50

20% of a number is 180. What is the number?

Answer

20% of a number is 180

1% of that number is $\frac{180}{20}$

Therefore 100% (all of it) of that number is $\frac{180}{20} \times 100$

$= 900$

Percentage increase or decrease

Note: The amount of increase (or decrease) is always expressed as a percentage of the original amount.

Example 51

If the population of a town decreased from 20 000 to 15 000, what was the percentage decrease?

Answer

Actual decrease = 20 000 − 15 000 = 5000

$$\text{Percentage decrease} = \frac{5000}{20\,000} \times 100$$

$$\text{We can now cancel above and below the line} = \frac{5\text{ØØØ}}{2\text{Ø}\,\text{ØØØ}} \times 10\text{Ø}$$

$= \frac{5}{2} \times 10 = \frac{50}{2} = 25\%$

Percentage decrease = 25%

Example 52

A school has 630 students in 1997. This was an increase of $12\frac{1}{2}\%$ on the previous year. How many students attended the school in 1996?

Answer

Number of students in 1997 = 630
630 is made up of number of students in 1996 plus $12\frac{1}{2}\%$ increase

$= 100\% + 12\frac{1}{2}\%$
$= 112\frac{1}{2}\%$

so $112\frac{1}{2}\% = 630$ students

$$1\% = \frac{630}{112\frac{1}{2}}$$

$$100\% = \frac{630}{112\frac{1}{2}} \times 100$$

$$= 560$$

$= 560$ students.

Exercise 1.17

Give your answers correct to two decimal places where necessary.

1. 18% of a number is 90. What is the number?
2. 25% of a number is 200. What is the number?
3. $2\frac{1}{2}\%$ of a number is 50. What is the number?
4. Increase 710 by 20%.
5. Increase 620 by 50%.
6. Decrease 280 by 15%.
7. A number is increased by 75%. If the new number is 8000, what was the original number?
8. A household commodity cost €400. The same product cost €430 a year later. What is the percentage increase?

Revision Exercise 1.18

Give your answers correct to two decimal places where necessary.

1. 50% of a number is 580. What is the number?
2. 14% of a sum of money is €60. What is the sum of money?
3. In a class of 40 students, 18 travel to school by bus. What percentage of the class is this?
4. Caroline obtained 80 marks out of a possible 150 in her business calculations examination. What was her percentage mark?
5. Mary earns €330 per week and saves €45 of this amount. What percentage of her money does she save? She spends 25% of her earnings on travel. Calculate the amount of money she spends on travel each week.
6. Patricia walks $\frac{1}{2}$ km to school and takes a bus for the remaining $2\frac{1}{2}$ km. What percentage of the total journey does she walk?